Spatial Cross-correlation Models for Vector Intensity Measures (PGA, Ia, PGV and Sa's) Considering Regional Site Conditions

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Outlines

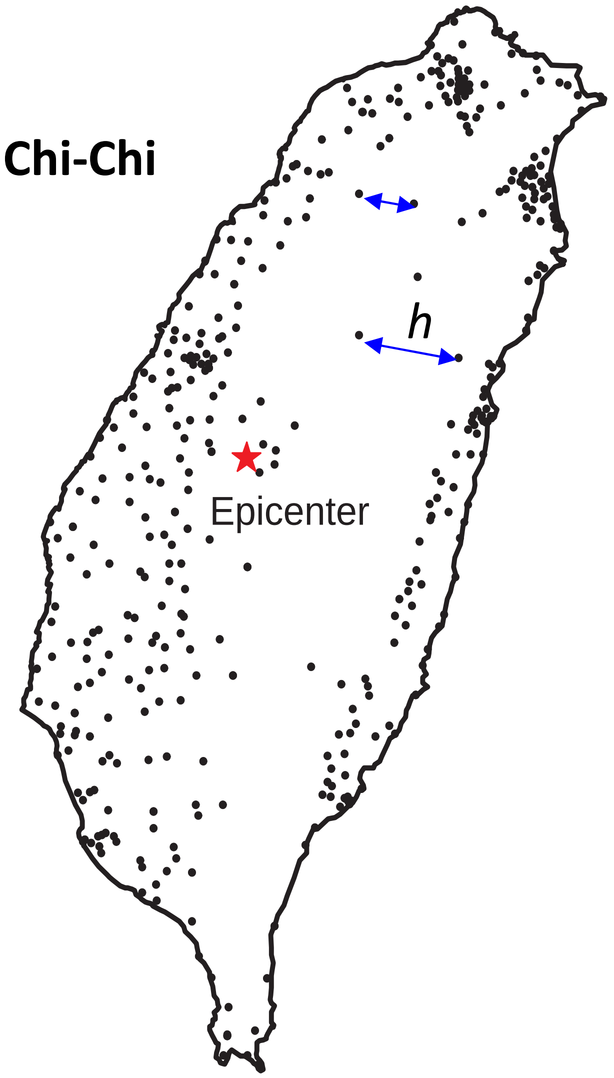
- Introduction
- Spatial Correlation of Scalar Intensity Measures
- Strong Motion Database and Regional Site Conditions
- Spatial Cross-correlation of Vector Intensity Measures
- A Site-dependent LMC Model for [PGA, Ia, PGV]
- A Site-dependent LMC Model for Sa(T)
- Applications and Conclusions

The presentation is based on
Wang and Du (2013), Spatial Cross-correlation Models for Vector Intensity Measures (PGA, Ia, PGV and Sa's)
Considering Regional Site Conditions, submitted to Bulletin of Seismological Society of America, under review.

Introduction

- Modeling spatial variability of ground-motion intensity measures (IMs) is essential for the seismic hazard analysis and risk assessment of spatially distributed infrastructure, such as lifelines, transportation systems and structure portfolios;
- Spatial correlation is not accounted for by GMPEs;
- It is necessary to consider the simultaneous occurrence of multiple intensity measure (Vector IM);
- It is important to consider the influence of regional geological features on the correlation structures.

Introduction



Epicenter and distribution of record stations for the Chi-Chi earthquake

Ground motion prediction equation:

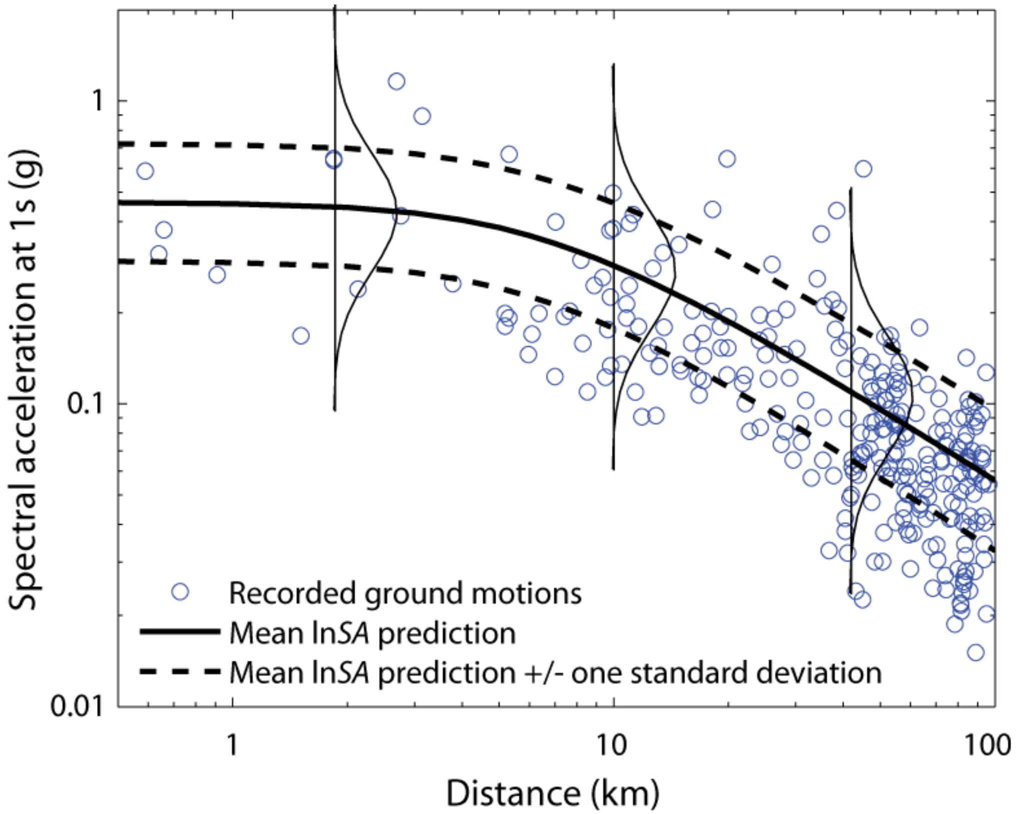
$$\ln Y_{ij} = \overline{\ln Y_{ij}\left(M,R,\theta\right)} + \eta_i + \varepsilon_{ij}$$

Measured data

Median prediction

Inter-event residual

Intra-event residual



Distribution of intra-event residuals (from Baker)

Spatial Correlation of Scalar Intensity Measures

- **A Missing Link – Spatial Correlation**

  The joint probability of occurrence of ground motion residuals in space.

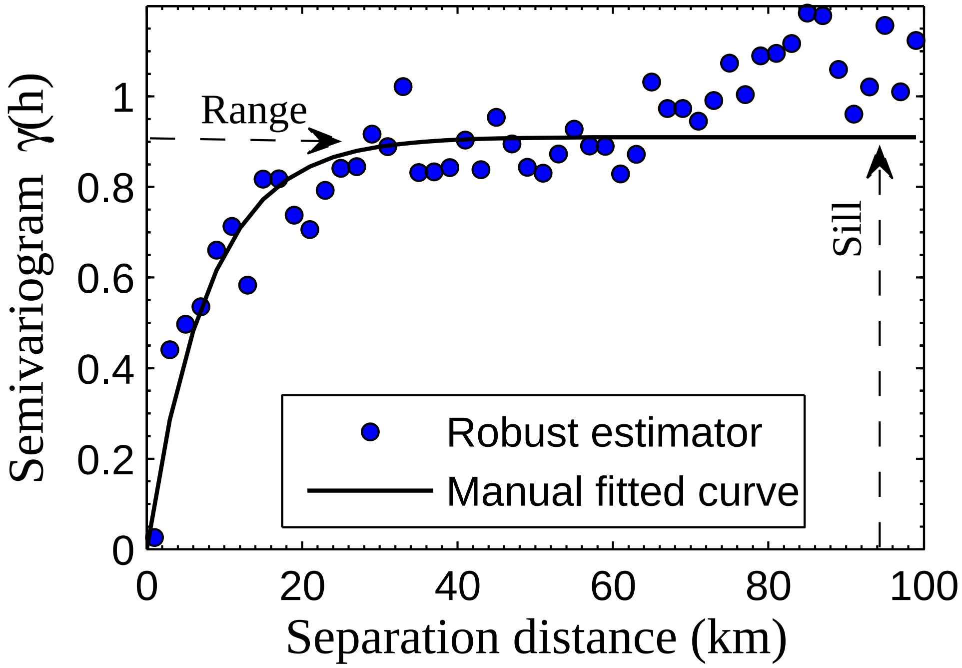
- **Empirical semi-variograms for intra-event residuals can be developed to measure the dissimilarity of data separated by separation distance *h*.**

$$\tilde{\gamma}(h)=\frac{1}{2\left|N(h)\right|}\sum_{i=1}^{N(h)}\left[z(u_i+h)-z(u_i)\right]^2$$

The closer the separation distance, the higher probability they are similar.

- **An exponential model can be used to fit the semivariograms.**

$$\boxed{\gamma(h)=a(1-\exp(-3h/b))}$$



***a*: the sill of correlation**
***b:* the range of correlation**
***h*: the separation distance**

$b = 24\ km$

Spatial Correlation of Scalar Intensity Measures

- **An exponential model can be used to fit the semivariograms.**

$$\boxed{\gamma(h) = a(1 - \exp(-3h/b))}$$

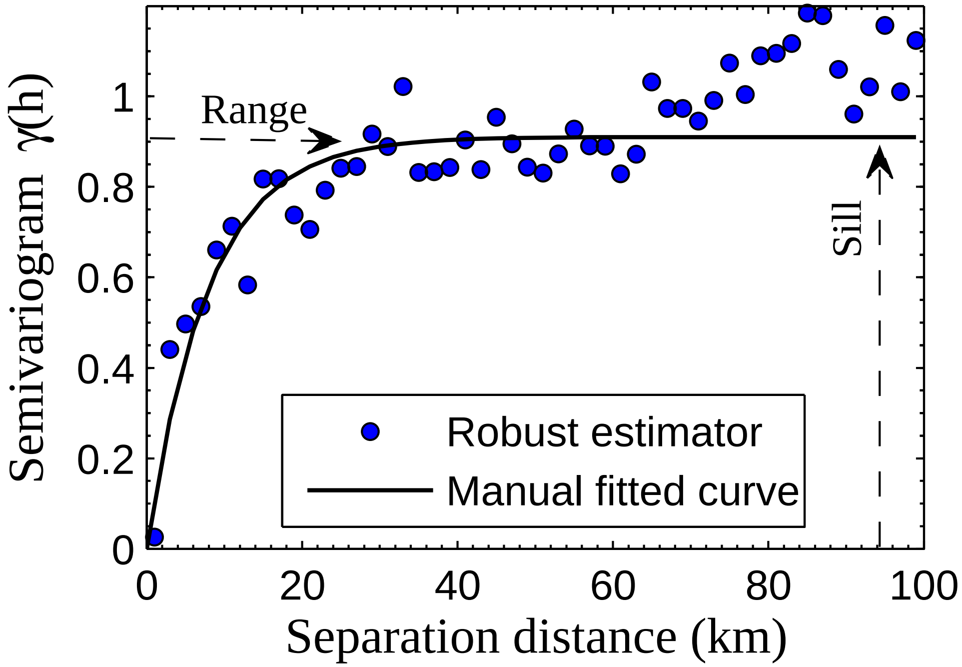
- **Spatial correlation coefficient**

$$\boxed{\rho(h) = \exp(-3h/b)}$$

- **The valid spatial correlation matrix is always (symmetric) positive semi-definite**

$$\begin{bmatrix} 1 & \rho(h_{12}) & \cdots & \rho(h_{1J}) \\ \rho(h_{12}) & 1 & & \\ \vdots & & \ddots & \\ \rho(h_{1J}) & \rho(h_{2J}) & \cdots & 1 \end{bmatrix} \geq 0$$

Valid correlation matrix



a: **the sill of correlation**
b: **the range of correlation**
h: **the separation distance**

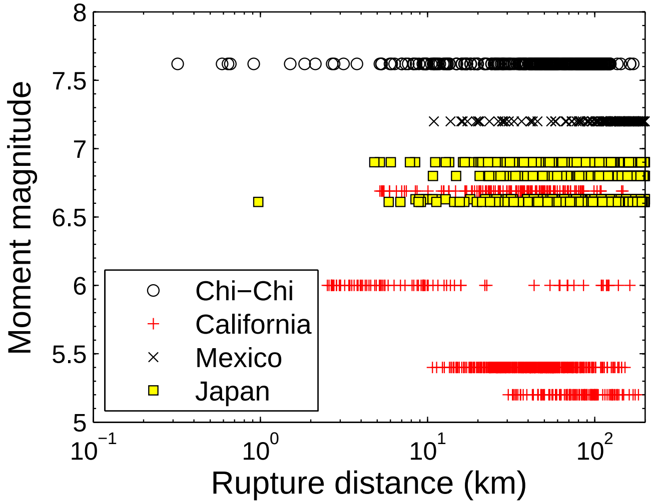
$$b = 24\ km$$

Strong Motion Database

Earthquake name	Date (dd/mm/yyyy)	Moment magnitude	Location	Fault mechanism	Num. of recordings
Northridge	01/17/1994	6.69	California	Reverse	152
Chi-Chi	09/20/1999	7.62	Taiwan	Reverse-oblique	401
Tottori	10/06/2000	6.61	Japan	Strike-slip	235
Parkfield	09/28/2004	6	California	Strike-slip	90
Niigata	10/23/2004	6.63	Japan	Reverse	365
Anza	06/12/2005	5.2	California	Reverse-oblique	111
Chuetsu	07/16/2007	6.8	Japan	Reverse	401
Alum Rock	10/30/2007	5.4	California	Strike-slip	161
Iwate	06/13/2008	6.9	Japan	Reverse	279
Chino Hills	07/29/2008	5.4	California	Reverse-oblique	337
EI Mayor Cucapah	04/04/2010	7.2	Mexico	Strike-slip	154

Note: only recorded data within rupture distance of 200 km are included for Japan earthquakes.

- **Eleven well-recorded earthquakes (2686 records) are used to investigate the spatial correlation of PGA, Ia, PGV and Sa(T).**

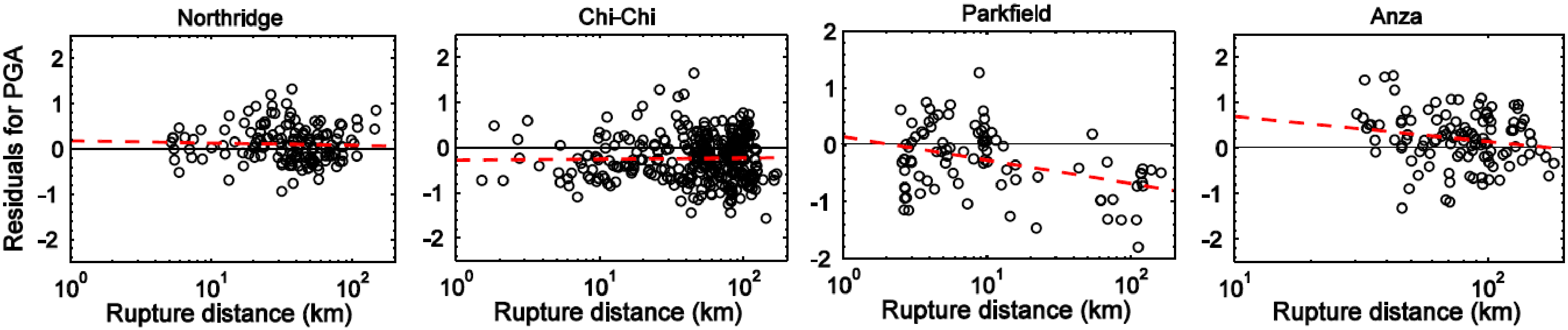


Magnitude and rupture distance distribution of records in the database.

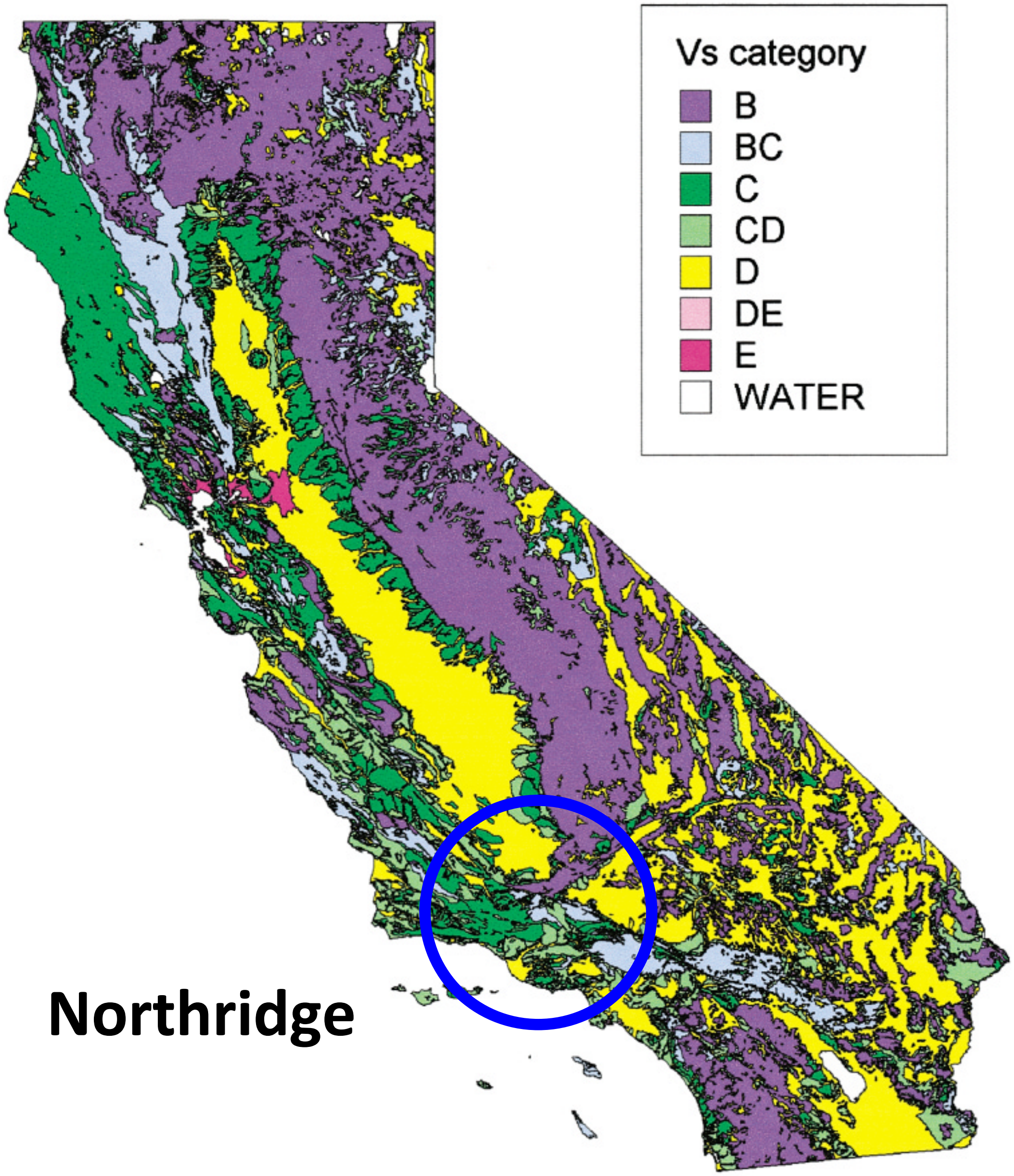
Strong Motion Database

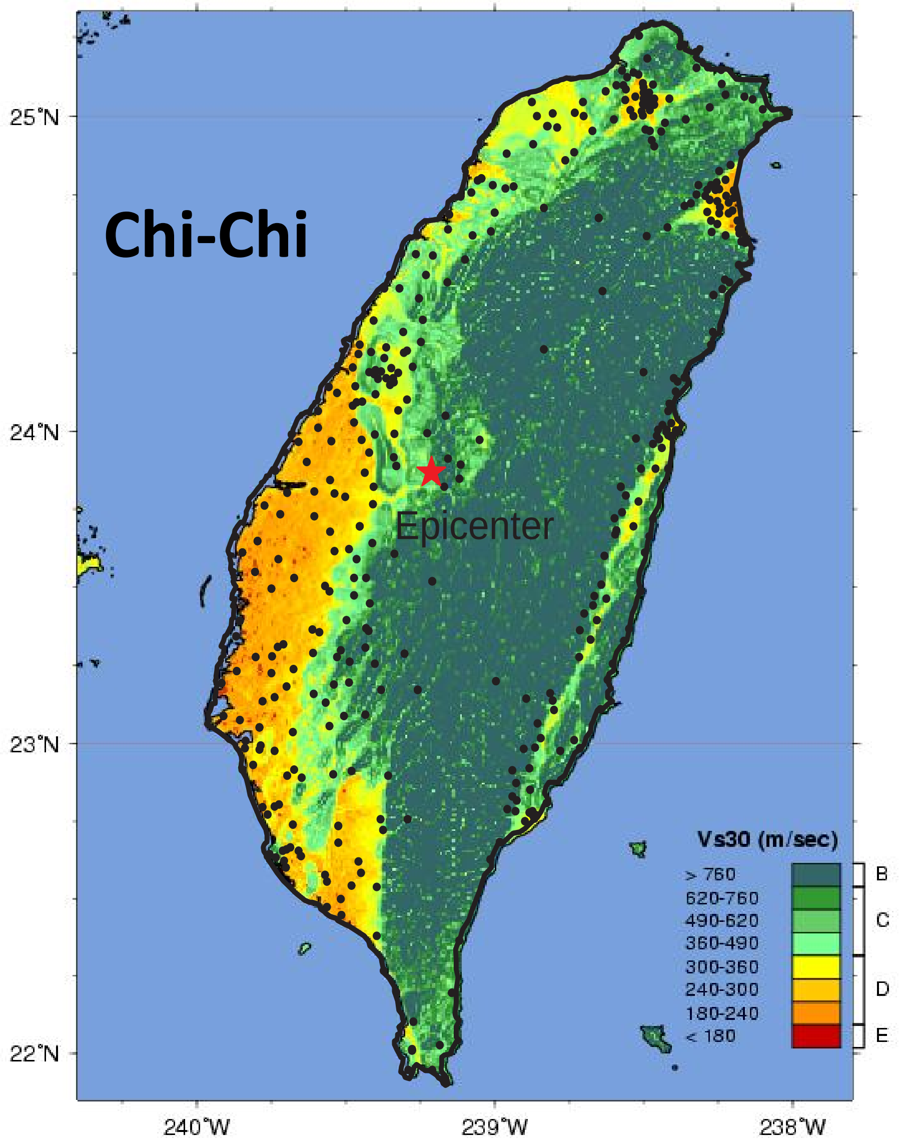
- **The trend of residuals versus rupture distance and Vs30 should be corrected to avoid artificial correlation.**

$$\varepsilon_{corr} = \ln Y_{ij} - \overline{\ln Y_{ij}\left(M, R, \theta\right)} - \boxed{\left(\varphi_1 + \varphi_2 \ln(R_{ij}) + \varphi_3 \ln(V_{s30})\right)}$$



Regional Site Conditions



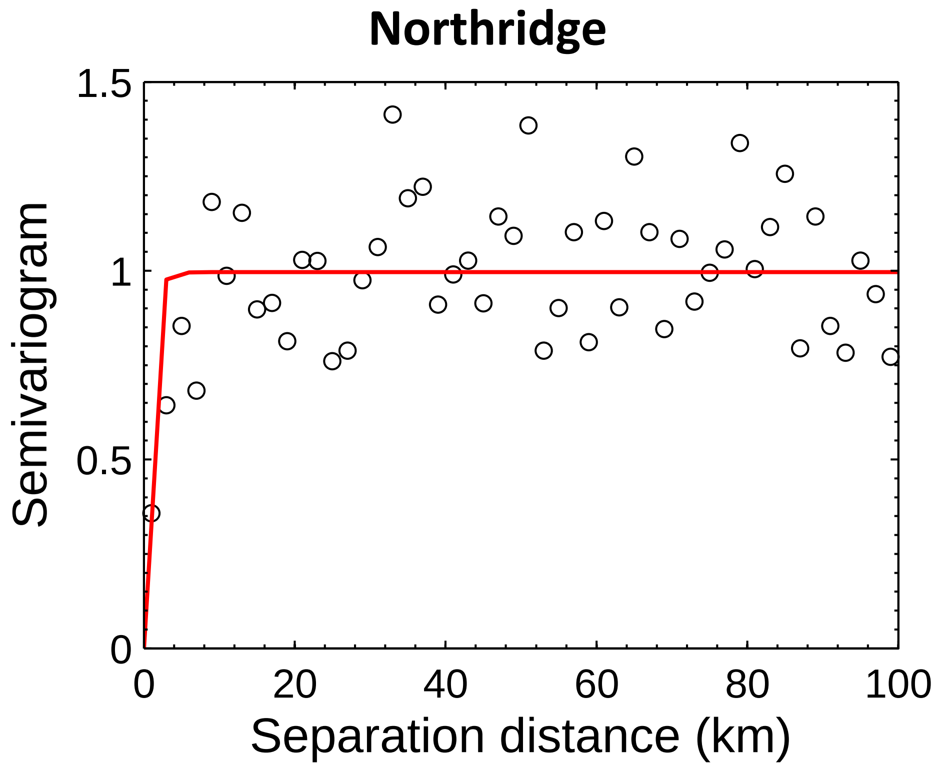


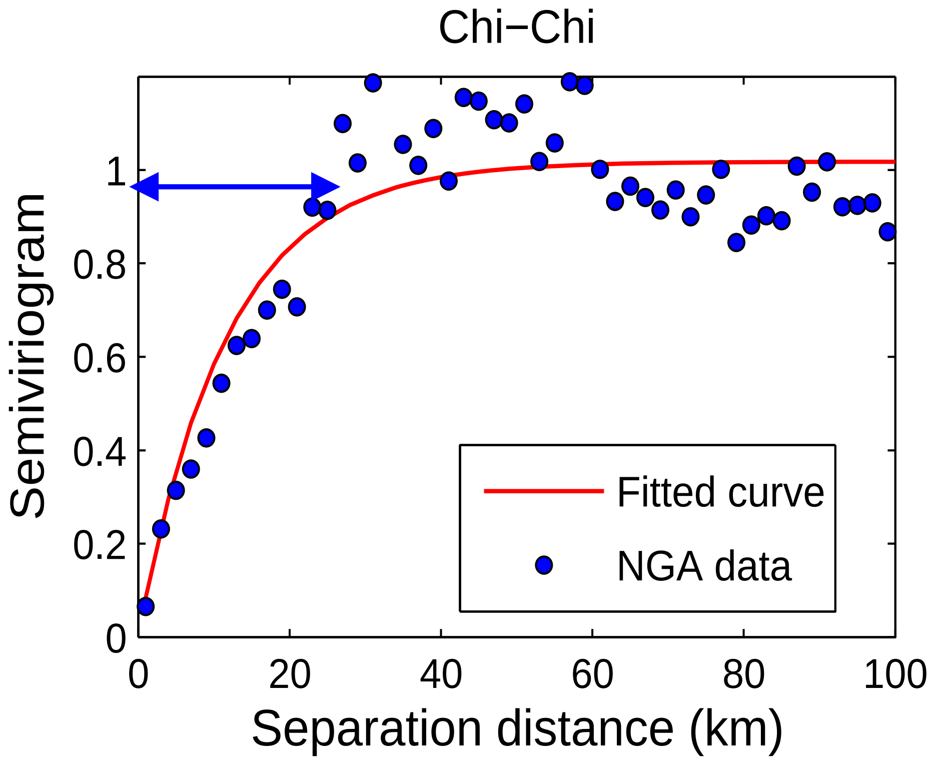
Heterogeneous

Homogeneous

Regional Site Conditions

The correlation range of normalized V_{s30} values (R_{Vs30}) are used to quantify the regional site conditions.


Northridge


Chi−Chi

R_{Vs30}= 0 km

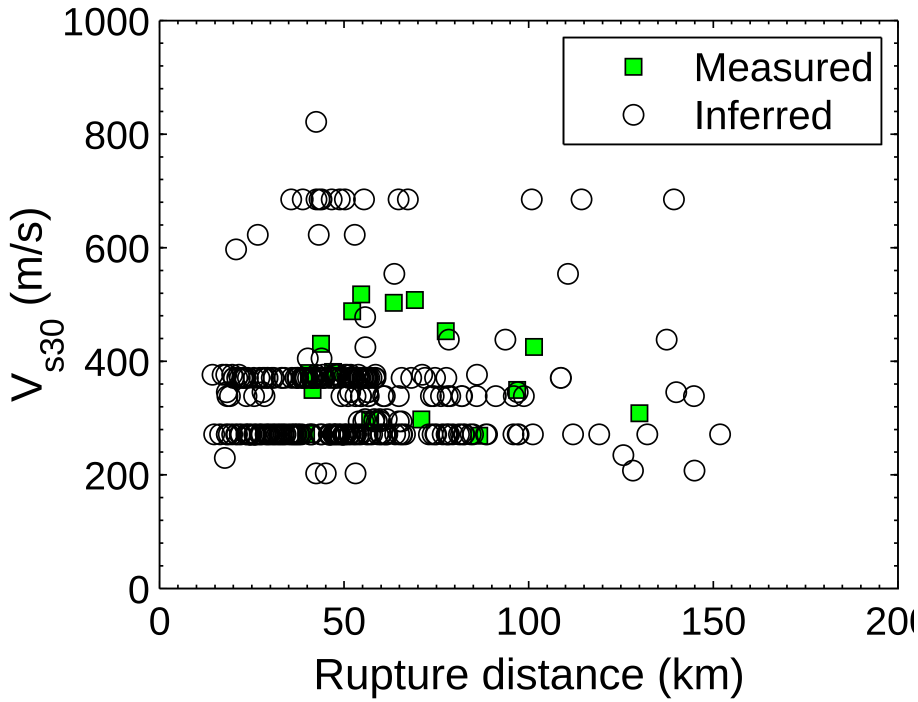
Heterogeneous

R_{Vs30}= 26 km

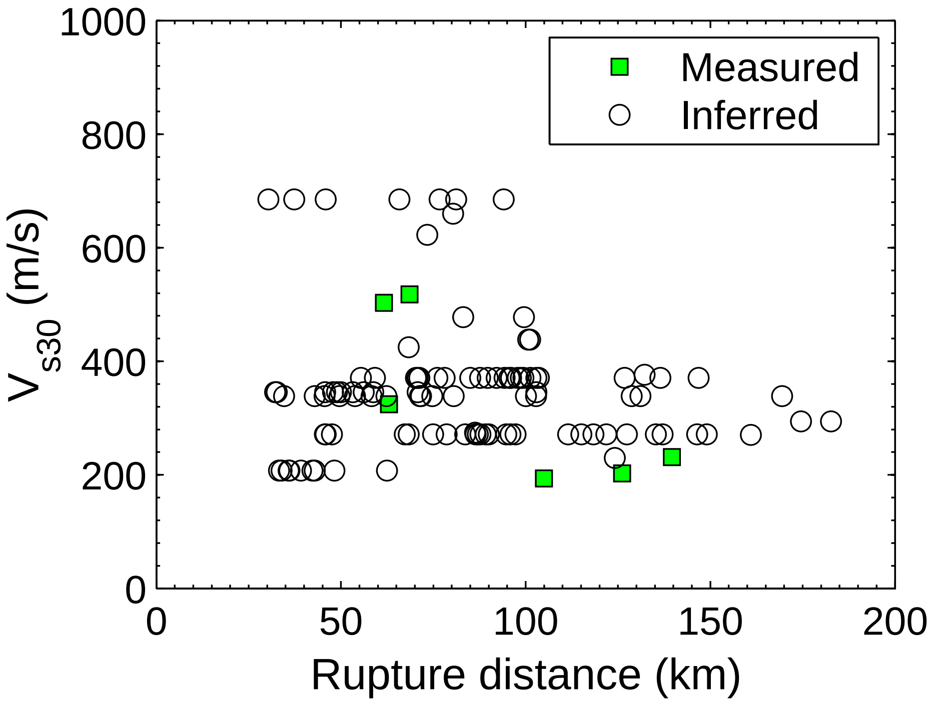
Homogeneous

Regional Site Conditions

- **Influence of inferred Vs30 data.**



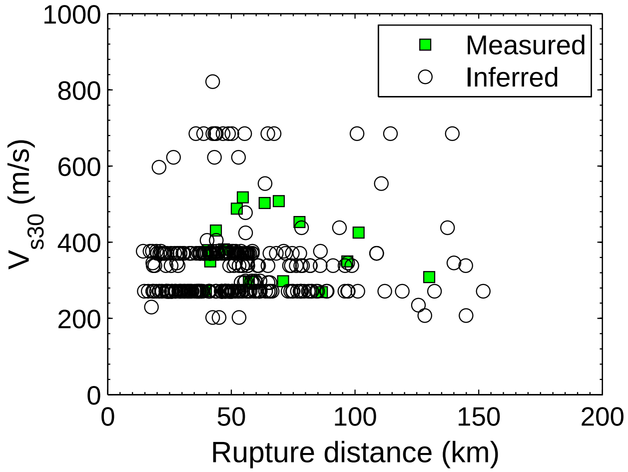
Anza earthquake



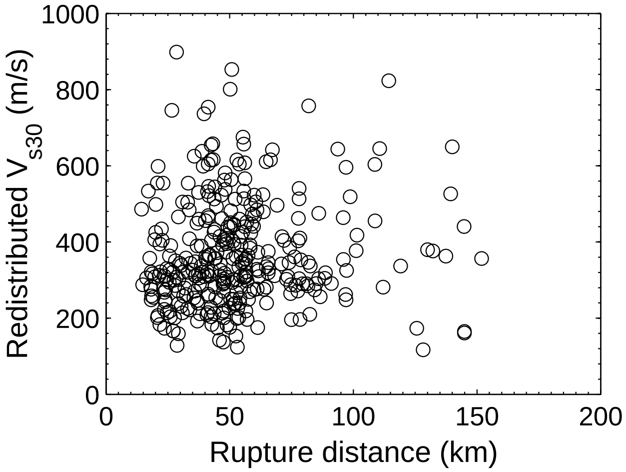
Chino Hills earthquake

- **A redistributed procedure is applied considering the uncertainty of Vs30 data to reduce artificial correlation induced by inferred data.**

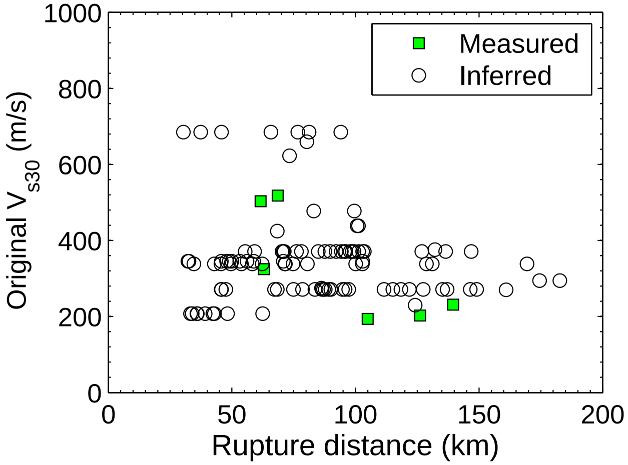
Regional Site Conditions



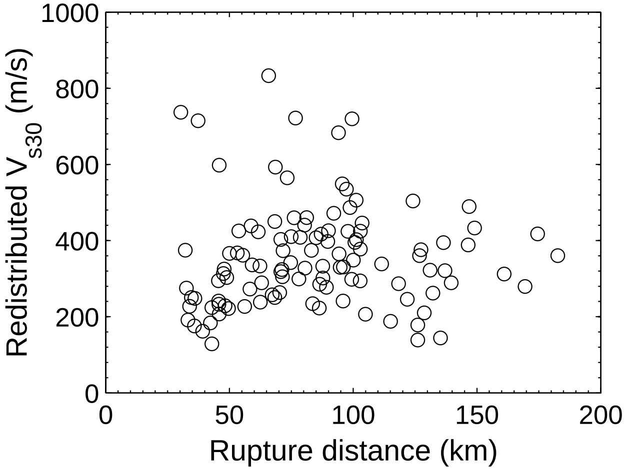
Anza earthquake



Randomized data

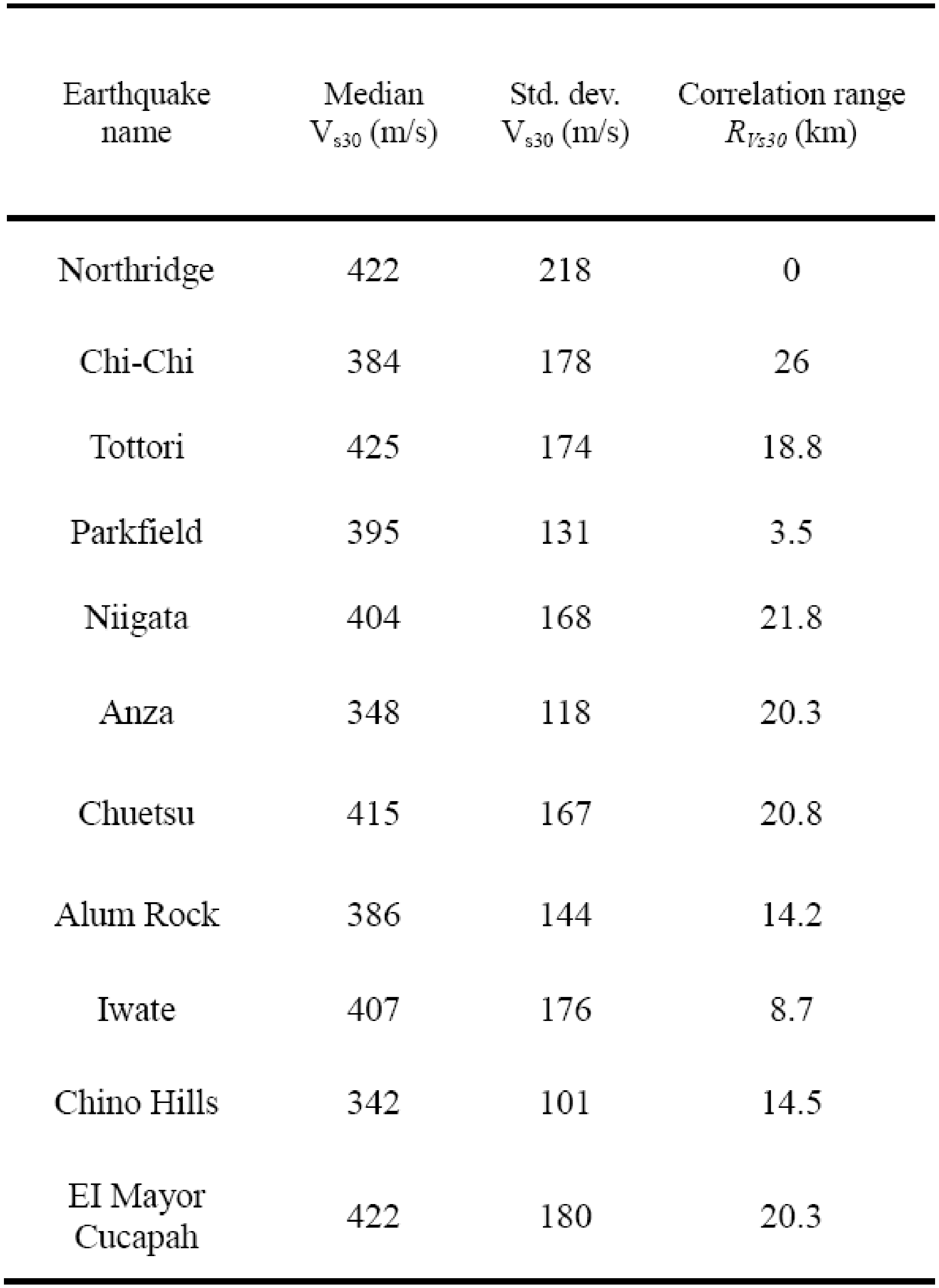


Chino Hills earthquake



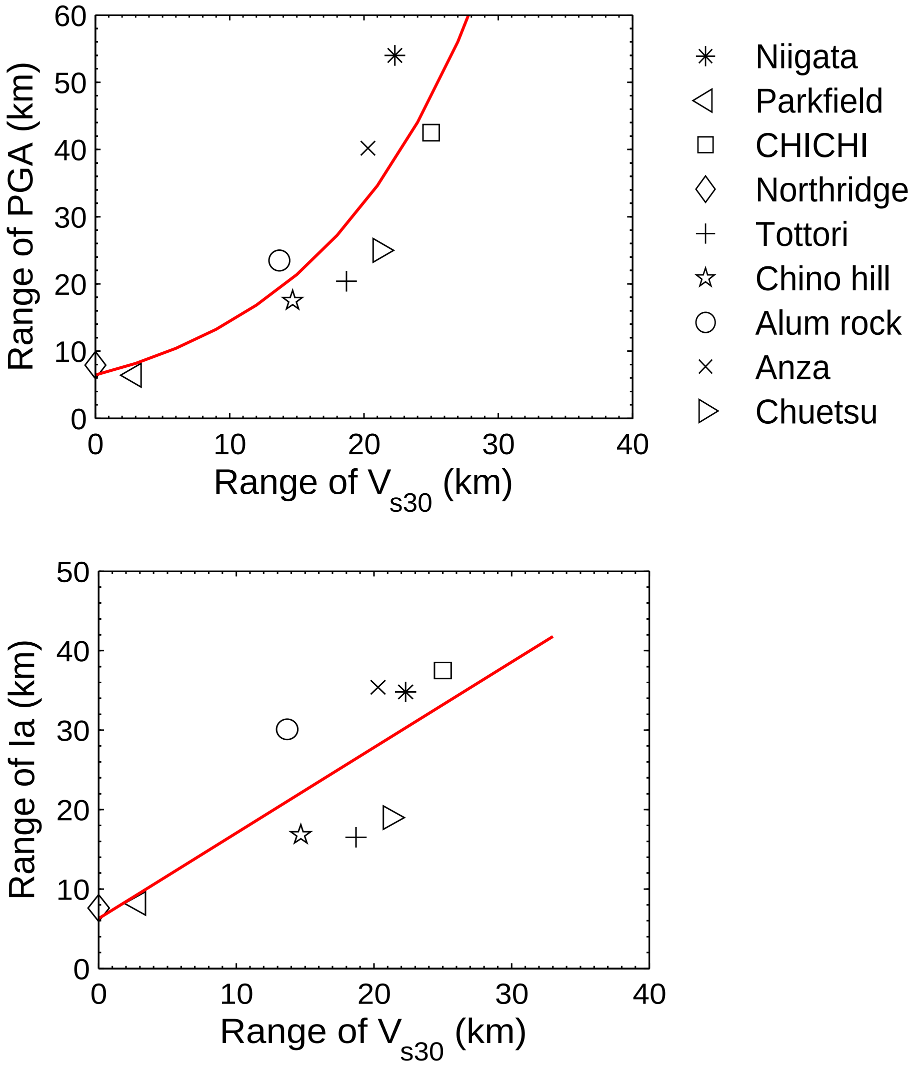
Randomized data

Regional Site Conditions



Earthquake name	Median V_{s30} (m/s)	Std. dev. V_{s30} (m/s)	Correlation range R_{Vs30} (km)
Northridge	422	218	0
Chi-Chi	384	178	26
Tottori	425	174	18.8
Parkfield	395	131	3.5
Niigata	404	168	21.8
Anza	348	118	20.3
Chuetsu	415	167	20.8
Alum Rock	386	144	14.2
Iwate	407	176	8.7
Chino Hills	342	101	14.5
EI Mayor Cucapah	422	180	20.3

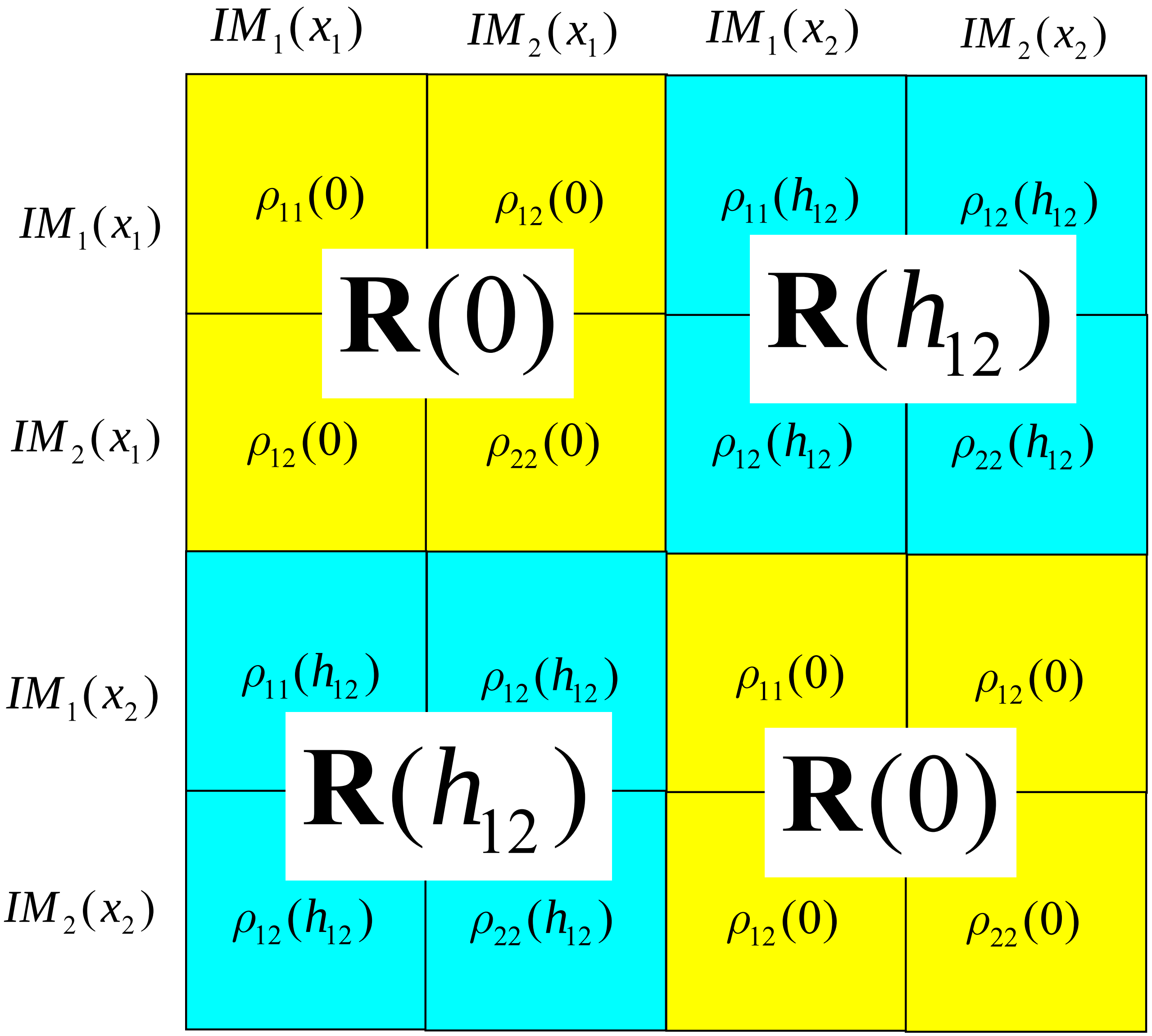
R_{Vs30} is closely related to the correlation range of scalar IMs



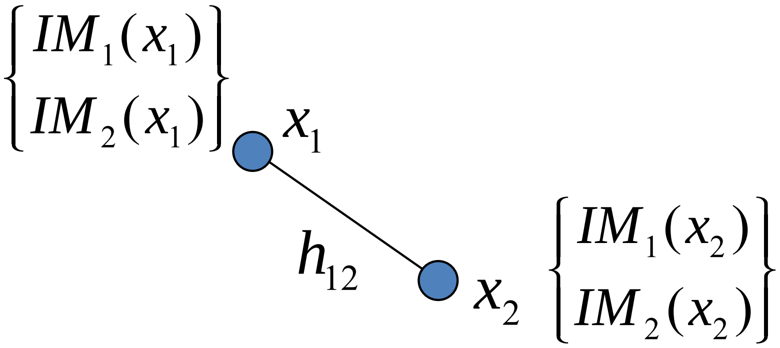
(Du and Wang, BSSA 2013, in press)

Spatial Cross-correlation of Vector Intensity Measures

Cross-correlation matrix



$$\begin{array}{c|cccc} & IM_1(x_1) & IM_2(x_1) & IM_1(x_2) & IM_2(x_2) \\ \hline IM_1(x_1) & \rho_{11}(0) & \rho_{12}(0) & \rho_{11}(h_{12}) & \rho_{12}(h_{12}) \\ IM_2(x_1) & \rho_{12}(0) & \rho_{22}(0) & \rho_{12}(h_{12}) & \rho_{22}(h_{12}) \\ IM_1(x_2) & \rho_{11}(h_{12}) & \rho_{12}(h_{12}) & \rho_{11}(0) & \rho_{12}(0) \\ IM_2(x_2) & \rho_{12}(h_{12}) & \rho_{22}(h_{12}) & \rho_{12}(0) & \rho_{22}(0) \end{array}$$



- Cross spatial correlation between *IM_1* and *IM_2* at separation distance *h_{12}*

- Given an *n*-component vector **IM** distributed at *J* sites, the total correlation matrix is [*Jxn*, *Jxn*] in dimension

$$\begin{bmatrix} \mathbf{R}(0) & \cdots & \mathbf{R}(h_{1J}) \\ \vdots & \ddots & \vdots \\ \mathbf{R}(h_{J1}) & \cdots & \mathbf{R}(0) \end{bmatrix}$$

Linear Model of Coregionalization for Vector IMs

- A combination of a short range and a long range exponential basic function are selected to fit empirical data (LMC)

$$\mathbf{R}(h) = \mathbf{P}^1\left(\exp\left(\frac{-3h}{r_1}\right)\right) + \mathbf{P}^2\left(\exp\left(\frac{-3h}{r_2}\right)\right)$$

- As long as **P¹** and **P²** are **positive semi-definite**, the total correlation matrix is guaranteed to be positive semi-definite regardless of sites considered (**a permissible/valid LMC model**).

$$\mathbf{P}^i = \begin{array}{cc} \begin{array}{ccc} \text{PGA} & \text{Ia} & \text{PGV} \end{array} & \\ \begin{bmatrix} \mathrm{p}_{11}^i & \mathrm{p}_{12}^i & \mathrm{p}_{13}^i \\ \mathrm{p}_{21}^i & \mathrm{p}_{22}^i & \mathrm{p}_{23}^i \\ \mathrm{p}_{31}^i & \mathrm{p}_{32}^i & \mathrm{p}_{33}^i \end{bmatrix} & \begin{array}{c} \text{PGA} \\ \text{Ia} \\ \text{PGV} \end{array} \end{array} \Longrightarrow \begin{bmatrix} \mathbf{R}(0) & \cdots & \mathbf{R}(h_{1J}) \\ \vdots & \ddots & \vdots \\ \mathbf{R}(h_{J1}) & \cdots & \mathbf{R}(0) \end{bmatrix}$$

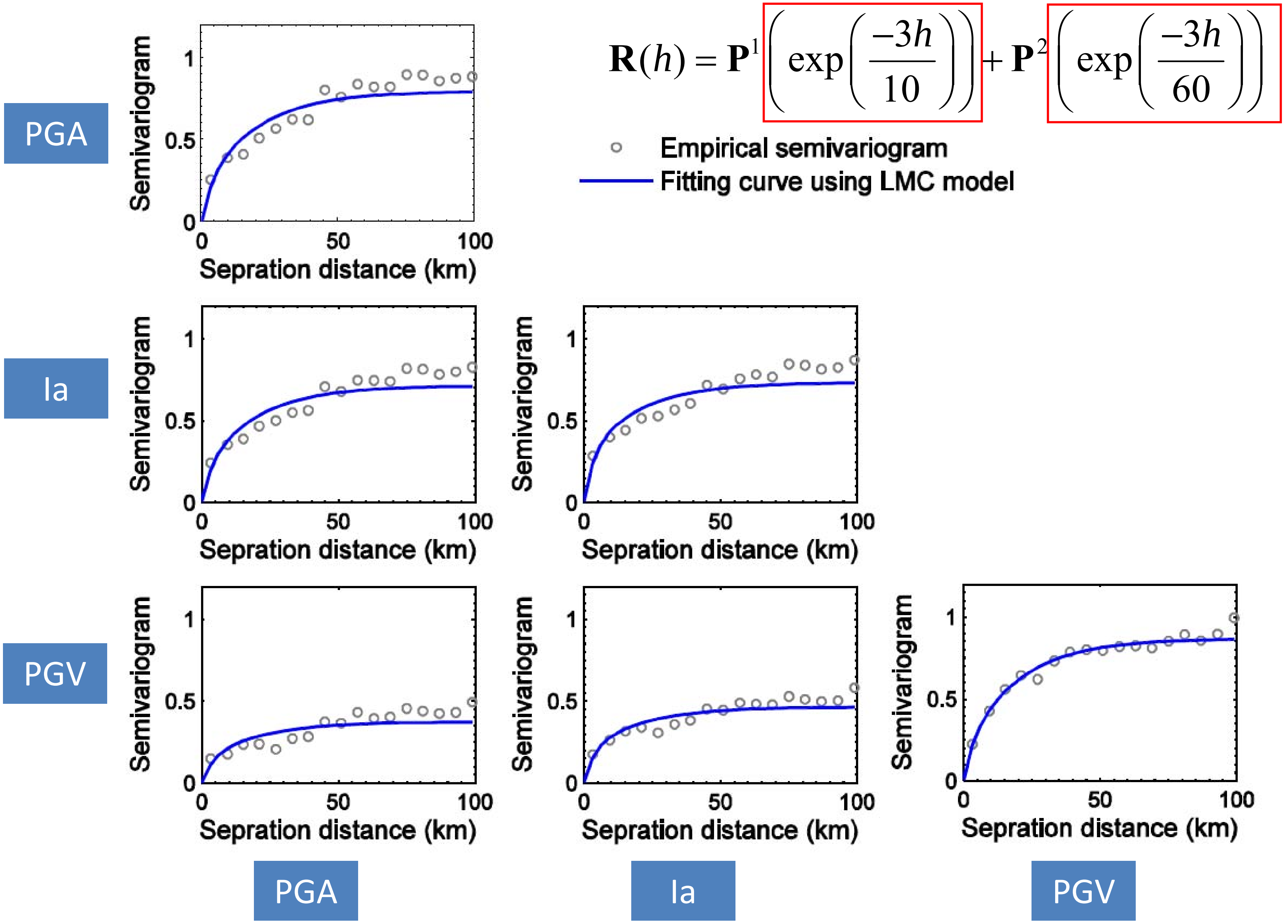
p.sd.

Valid correlation matrix

- The **P¹** and **P²** matrices can be obtained from each earthquake

Linear Model of Coregionalization for Vector IMs

$$\mathbf{R}(h) = \mathbf{P}^1 \boxed{\left(\exp\left(\frac{-3h}{10}\right)\right)} + \mathbf{P}^2 \boxed{\left(\exp\left(\frac{-3h}{60}\right)\right)}$$



Cross-semivariograms and fitted LMC curves obtained for the Niigata earthquake

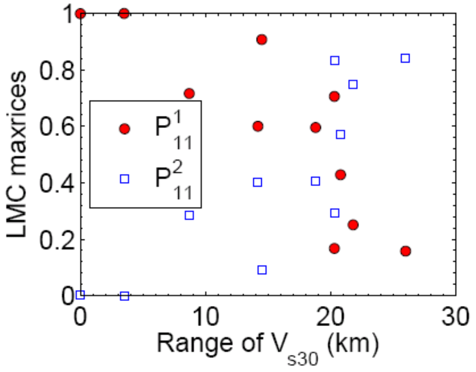
Influence of Site Condition on LMC Matrices

$$\mathbf{R}(h)=\boxed{\mathbf{P}^1}\left(\exp\left(\frac{-3h}{10}\right)\right)+\boxed{\mathbf{P}^2}\left(\exp\left(\frac{-3h}{60}\right)\right)$$

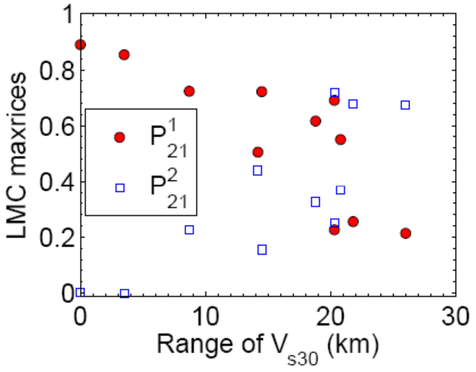
Short range Long range

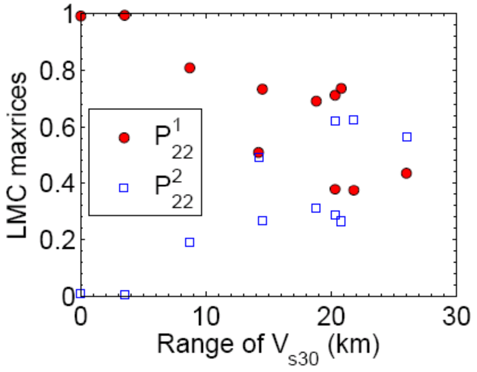
$$\mathbf{P}^i=\begin{bmatrix}\enclose{circle}{\mathrm{p}^i_{11}} & \mathrm{p}^i_{12} & \mathrm{p}^i_{13}\\ \enclose{circle}{\mathrm{p}^i_{21}} & \enclose{circle}{\mathrm{p}^i_{22}} & \mathrm{p}^i_{23}\\ \enclose{circle}{\mathrm{p}^i_{31}} & \enclose{circle}{\mathrm{p}^i_{32}} & \enclose{circle}{\mathrm{p}^i_{33}}\end{bmatrix}$$

PGA

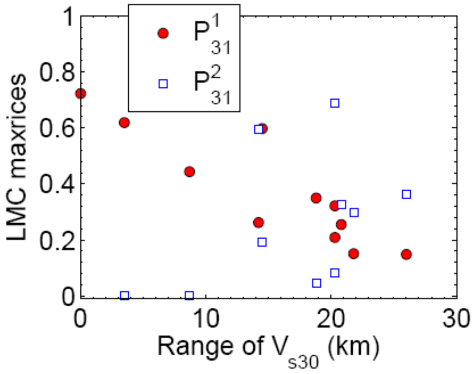


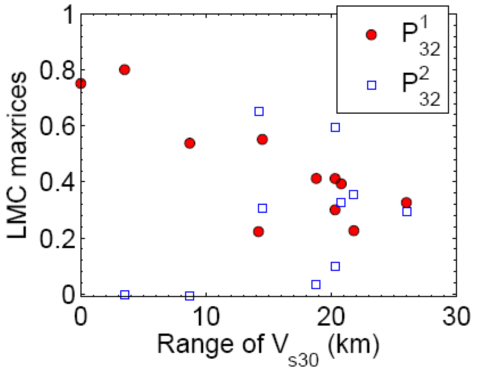
Ia

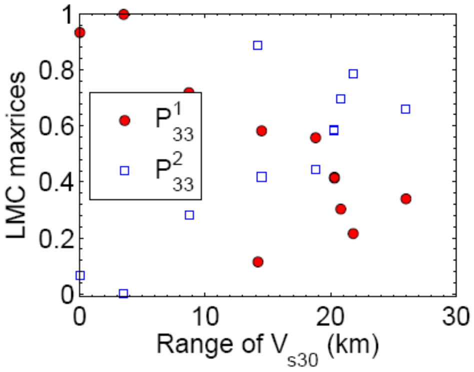




PGV



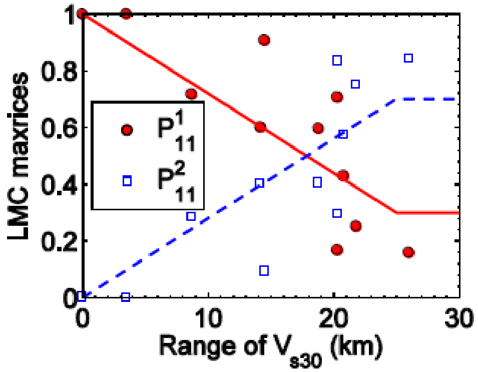




PGA **Ia** **PGV**

Influence of Site Condition on LMC Matrices

PGA

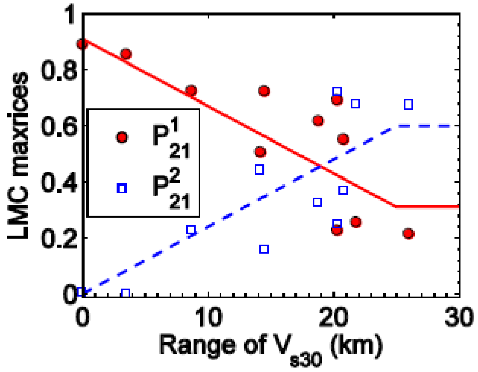


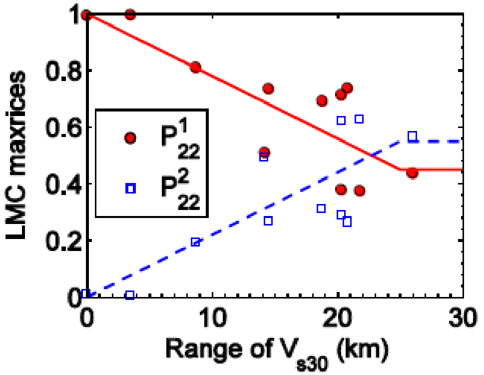
$$\mathbf{P}^1 = \mathbf{P}^0 - \mathbf{K} \cdot R_{Vs30}$$
$$\mathbf{P}^2 = \mathbf{K} \cdot R_{Vs30}$$

$$\mathbf{K} = \begin{bmatrix} 0.28 & 0.24 & 0.17 \\ 0.24 & 0.22 & 0.16 \\ 0.17 & 0.16 & 0.31 \end{bmatrix}$$

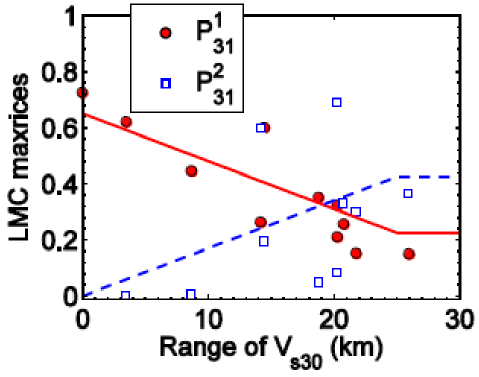
$$\mathbf{P}^0 = \begin{bmatrix} 1 & 0.91 & 0.65 \\ 0.91 & 1 & 0.71 \\ 0.65 & 0.71 & 1 \end{bmatrix}$$

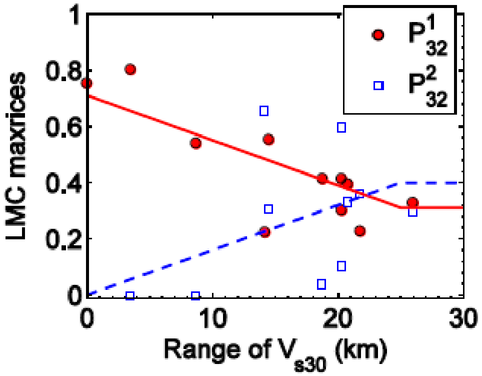
Ia

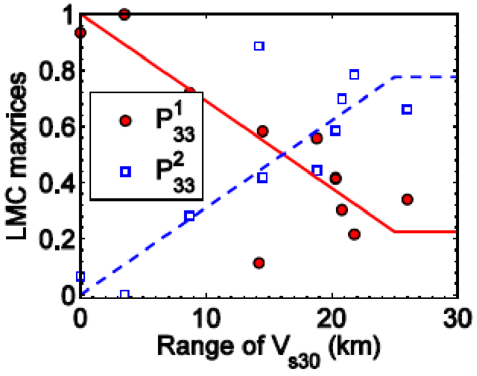




PGV







PGA

Ia

PGV

A Site-dependent LMC Model for [PGA, Ia, PGV]

- **Site-dependent LMC model**

$$\mathbf{R}(h, R_{Vs30}) = \mathbf{P}^1(R_{Vs30})\left(\exp\left(\frac{-3h}{10}\right)\right) + \mathbf{P}^2(R_{Vs30})\left(\exp\left(\frac{-3h}{60}\right)\right)$$

A permissible LMC model

$$\mathbf{P}^1 = \mathbf{P}^0 - \mathbf{K}\left(\frac{R_{Vs30}}{10}\right)$$

Positive definite for $R_{vs30} \leq 25$ km

$$\mathbf{P}^2 = \mathbf{K}\left(\frac{R_{Vs30}}{10}\right)$$

Positive definite

$$\mathbf{K} = \begin{bmatrix} 0.28 & 0.24 & 0.17 \\ 0.24 & 0.22 & 0.16 \\ 0.17 & 0.16 & 0.31 \end{bmatrix} \qquad \mathbf{P}^0 = \begin{bmatrix} 1 & 0.91 & 0.65 \\ 0.91 & 1 & 0.71 \\ 0.65 & 0.71 & 1 \end{bmatrix}$$

Positive definite

A Site-dependent LMC Model for [PGA, Ia, PGV]

- **Site-dependent LMC model**

$$\mathbf{R}(h,R_{Vs30}) = \boxed{\left[\mathbf{P}^0 - \mathbf{K}\left(\frac{R_{Vs30}}{10}\right)\right]}\left(\exp\left(\frac{-3h}{10}\right)\right) + \boxed{\mathbf{K}\left(\frac{R_{Vs30}}{10}\right)}\left(\exp\left(\frac{-3h}{60}\right)\right)$$

- **Examples**

$$\mathbf{R}(h=5,R_{Vs30}=20) = \begin{array}{cccc} \text{PGA} & \text{Ia} & \text{PGV} & \\ \left[\begin{matrix} 0.53 \\ 0.47 \\ 0.33 \end{matrix}\right. & \begin{matrix} 0.47 \\ 0.47 \\ 0.34 \end{matrix} & \left.\begin{matrix} 0.33 \\ 0.34 \\ 0.57 \end{matrix}\right] & \begin{matrix} \text{PGA} \\ \text{Ia} \\ \text{PGV} \end{matrix} \end{array}$$

$$\mathbf{R}(h=5,R_{Vs30}=10) = \begin{bmatrix} 0.38 & 0.34 & 0.24 \\ 0.34 & 0.35 & 0.25 \\ 0.24 & 0.25 & 0.40 \end{bmatrix}$$

Influence of site conditions

$$\mathbf{K} = \begin{bmatrix} 0.28 & 0.24 & 0.17 \\ 0.24 & 0.22 & 0.16 \\ 0.17 & 0.16 & 0.31 \end{bmatrix}$$

Weaker

Stronger

A Site-dependent LMC Model for [PGA, Ia, PGV]

- **Site-dependent LMC model**

$$\mathbf{R}(h, R_{Vs30}) = \boxed{\left[\mathbf{P}^{0} - \mathbf{K}\left(\frac{R_{Vs30}}{10}\right)\right]}\left(\exp\left(\frac{-3h}{10}\right)\right) + \boxed{\mathbf{K}\left(\frac{R_{Vs30}}{10}\right)}\left(\exp\left(\frac{-3h}{60}\right)\right)$$

- **Reduce to local correlation matrix (h=0)**

$$\mathbf{R}(h=0) \triangleq \mathbf{R}(0) = \mathbf{P}^{0} = \begin{bmatrix} 1 & 0.91 & 0.65 \\ 0.91 & 1 & 0.71 \\ 0.65 & 0.71 & 1 \end{bmatrix}$$

Compare with Campbell and Bozorgnia (2012):

$$\begin{bmatrix} \rho_{PGA,PGA} & \rho_{PGA,Ia} & \rho_{PGA,PGV} \\ \rho_{PGA,Ia} & \rho_{Ia,Ia} & \rho_{Ia,PGV} \\ \rho_{PGA,PGV} & \rho_{Ia,PGV} & \rho_{PGV,PGV} \end{bmatrix} = \begin{bmatrix} 1 & 0.88 & 0.69 \\ 0.88 & 1 & 0.74 \\ 0.69 & 0.74 & 1 \end{bmatrix}$$

A Site-dependent LMC Model for [PGA, Ia, PGV]

- **Site-dependent LMC model**

$$\mathbf{R}(h, R_{Vs30}) = \boxed{\left[\mathbf{P}^0 - \mathbf{K}\left(\frac{R_{Vs30}}{10}\right)\right]}\left(\exp\left(\frac{-3h}{10}\right)\right) + \boxed{\mathbf{K}\left(\frac{R_{Vs30}}{10}\right)}\left(\exp\left(\frac{-3h}{60}\right)\right)$$

- **Reduce to heterogeneous site conditions (R_{Vs30}=0)**

,

$$\mathbf{R}(h) = \mathbf{R}(0)\exp\left(\frac{-3h}{10}\right)$$

- **Averaged LMC model for [PGA, Ia, PGV]**

$$\mathbf{R}(h) = \mathbf{P}^1_{avg}\left(\exp\left(\frac{-3h}{10}\right)\right) + \mathbf{P}^2_{avg}\left(\exp\left(\frac{-3h}{60}\right)\right)$$

$$\mathbf{P}^1_{avg} = \begin{bmatrix} 0.61 & 0.57 & 0.38 \\ 0.57 & 0.67 & 0.45 \\ 0.38 & 0.45 & 0.50 \end{bmatrix} \qquad \mathbf{P}^2_{avg} = \begin{bmatrix} 0.39 & 0.34 & 0.24 \\ 0.34 & 0.33 & 0.24 \\ 0.24 & 0.24 & 0.50 \end{bmatrix}$$

Model Predictions -- Northridge Earthquake

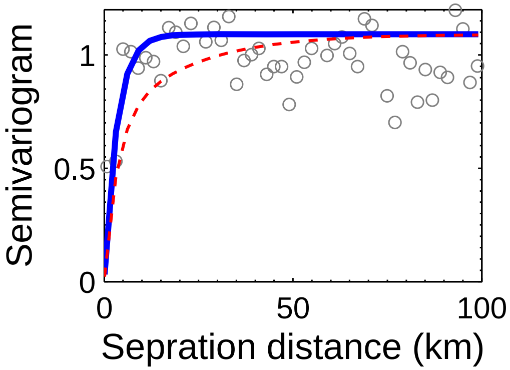
$$\mathbf{R}(h) = \mathbf{P}^1\left(\exp\left(\frac{-3h}{10}\right)\right) + \mathbf{P}^2\left(\exp\left(\frac{-3h}{60}\right)\right)$$

○ Empirical semivariogram

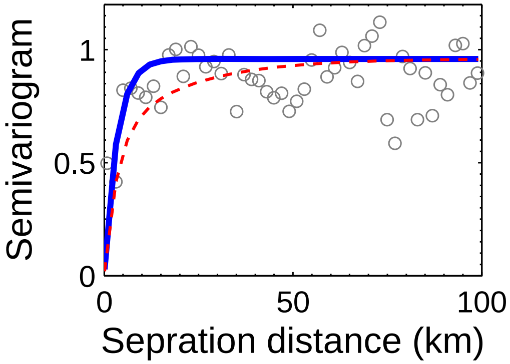
━━ Predicted curve using site−dependent matrices

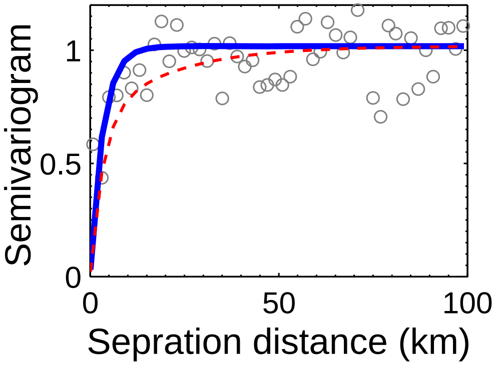
- - - Predicted curve using averaged matrices

PGA

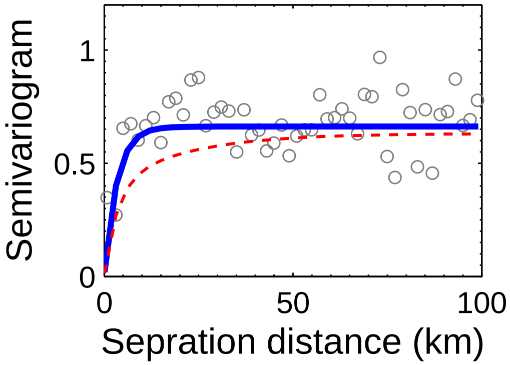


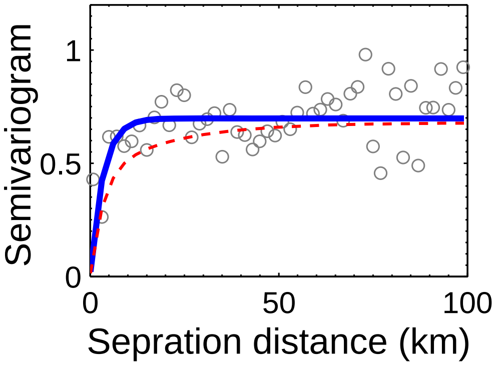
Ia

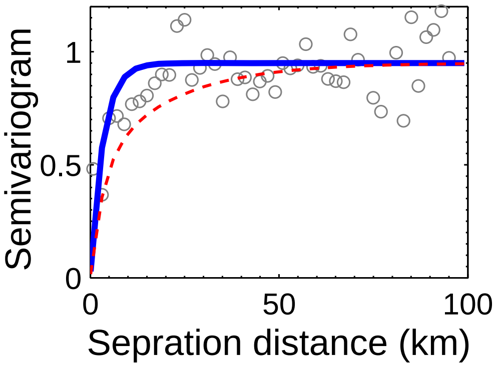




PGV







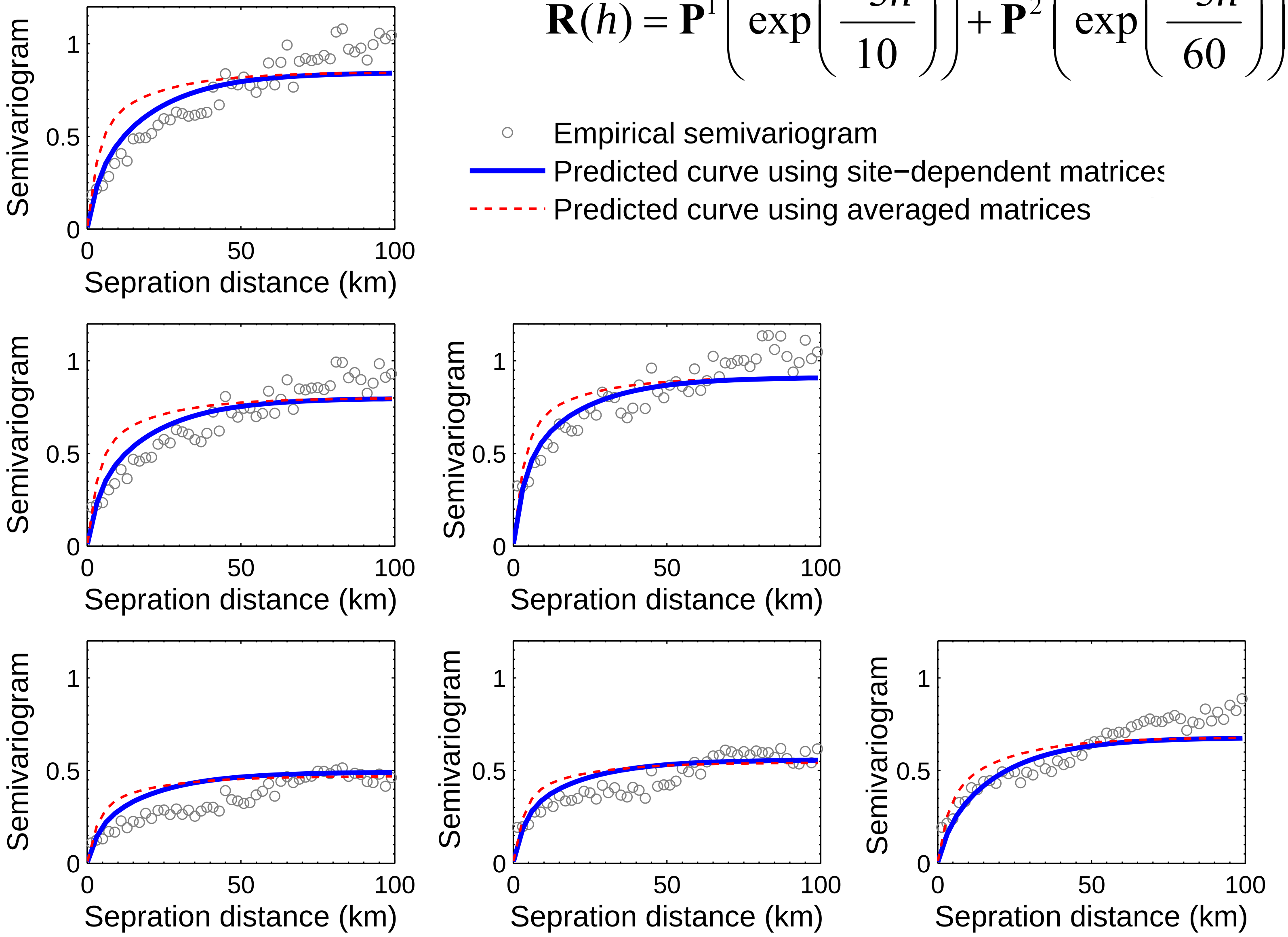
PGA

Ia

PGV

Model Predictions – Chi-Chi Earthquake

$$\mathbf{R}(h) = \mathbf{P}^1\left(\exp\left(\frac{-3h}{10}\right)\right) + \mathbf{P}^2\left(\exp\left(\frac{-3h}{60}\right)\right)$$



PGA

Ia

PGV

PGA

Ia

PGV

A Site-dependent LMC Model for Sa(T)

- **Site-dependent LMC model for Sa(T):**

$$\mathbf{R}(h, R_{Vs30}) = \mathbf{P}^1\left(\exp\left(\frac{-3h}{10}\right)\right) + \mathbf{P}^2\left(\exp\left(\frac{-3h}{70}\right)\right)$$

$$\mathbf{P}^1 = \boxed{\mathbf{P}_{Sa}^{01}} - \boxed{\mathbf{K}_{Sa}}\left(\frac{R_{Vs30}}{10}\right)$$

$$\mathbf{P}^2 = \boxed{\mathbf{P}_{Sa}^{02}} + \mathbf{K}_{Sa}\left(\frac{R_{Vs30}}{10}\right)$$

- **Averaged LMC model (Loth and Baker, 2013)**

$$\mathbf{R}(h) = \boxed{\mathbf{P}^1}\left(\exp\left(\frac{-3h}{20}\right)\right) + \boxed{\mathbf{P}^2}\left(\exp\left(\frac{-3h}{70}\right)\right) + \boxed{\mathbf{P}^3} I_{h=0}$$

Loth C., and J. W. Baker (2013). A spatial cross-correlation model of ground motion spectral accelerations at multiple periods. *Earthquake Engineering & Structural Dynamics*, (in press).

A Site-dependent LMC Model for Sa(T)

Period (s)	0.01	0.1	0.2	0.5	1	2	5	7.5	10
0.01	0.96								
0.1	0.9	0.96							
0.2	0.8	0.81	0.93				symmetric		
0.5	0.5	0.36	0.44	0.76					
1	0.15	0.08	0.1	0.25	0.62				
2	0.09	0.04	0.05	0.17	0.45	0.54			
5	0.1	0.05	0.09	0.14	0.34	0.42	0.47		
7.5	0.09	0.05	0.08	0.13	0.37	0.42	0.46	0.57	
10	0.04	0.02	0.05	0.07	0.31	0.35	0.39	0.4	0.56

$\mathbf{P}_{Sa}^{01}$

p.d.

$$\mathbf{P}^{1} = \mathbf{P}_{Sa}^{01} - \mathbf{K}_{Sa}\left(\frac{R_{Vs30}}{10}\right)$$

Positive definite for R_{vs30}≤25 km

+

Period (s)	0.01	0.1	0.2	0.5	1	2	5	7.5	10
0.01	0.04								
0.1	0	0.04							
0.2	0.01	0.01	0.07				symmetric		
0.5	0.04	0	0.08	0.24					
1	0.08	0.01	0.08	0.28	0.38				
2	0.02	0	0.01	0.2	0.22	0.46			
5	0.02	0	0	0.15	0.23	0.32	0.53		
7.5	0	0	0	0.13	0.18	0.25	0.43	0.43	
10	0.02	0	0	0.13	0.19	0.25	0.42	0.41	0.44

$\mathbf{P}_{Sa}^{02}$

p.d.

$$\mathbf{P}^{2} = \mathbf{P}_{Sa}^{02} + \mathbf{K}_{Sa}\left(\frac{R_{Vs30}}{10}\right)$$

Positive definite

Period (s)	0.01	0.1	0.2	0.5	1	2	5	7.5	10
0.01	0.28								
0.1	0.26	0.27							
0.2	0.2	0.21	0.2				symmetric		
0.5	0.13	0.1	0.1	0.11					
1	0	0	0	0	0.14				
2	0	0	0	0	0.11	0.11			
5	0	0	0	0	0.08	0.09	0.11		
7.5	0	0	0	0	0.1	0.11	0.12	0.14	
10	0	0	0	0	0.1	0.12	0.12	0.13	0.17

$\mathbf{K}_{Sa}$

p.d.

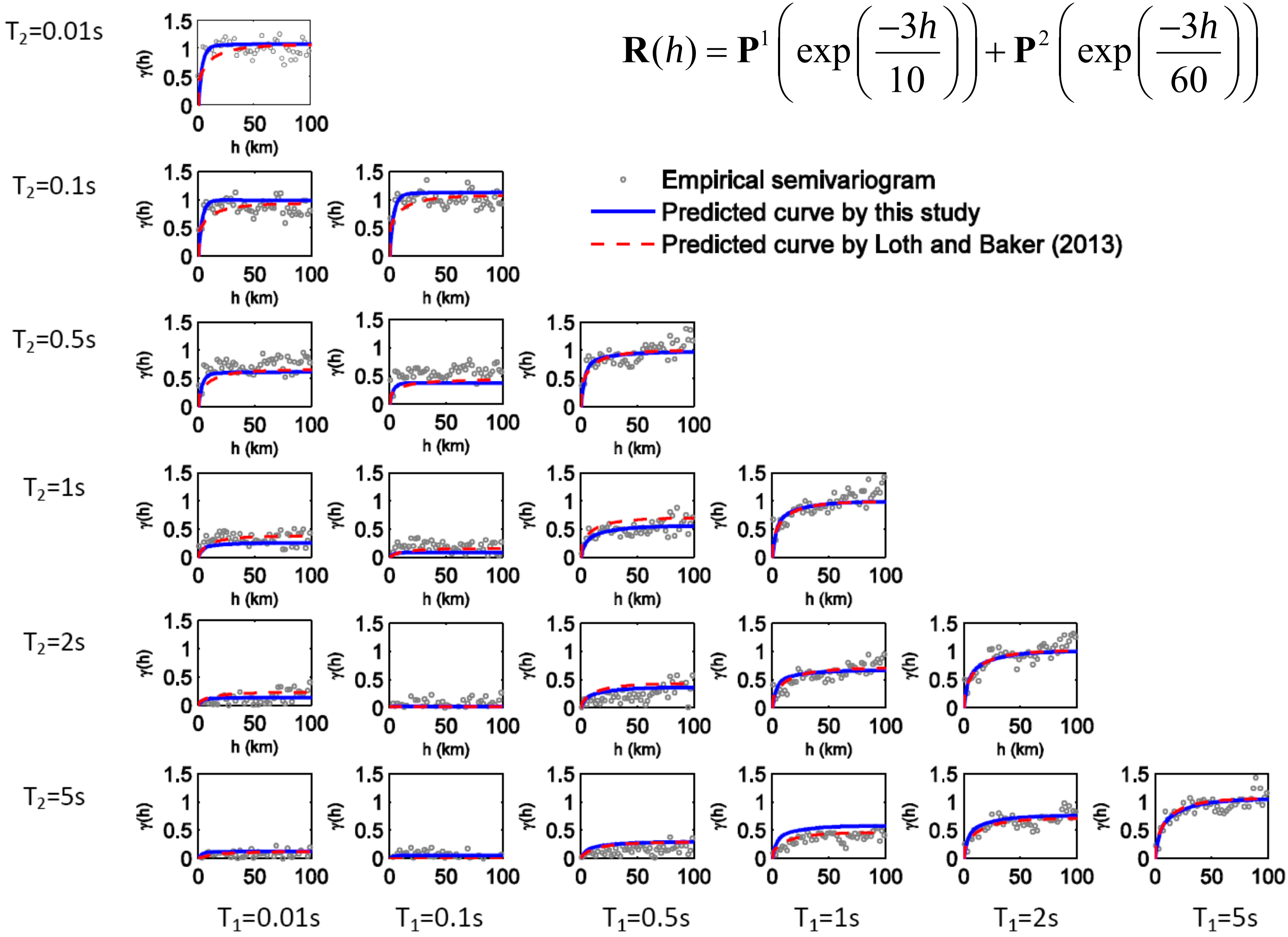
$$\mathbf{R} = \mathbf{P}^{1}\left(\exp\left(\frac{-3h}{10}\right)\right) + \mathbf{P}^{2}\left(\exp\left(\frac{-3h}{70}\right)\right)$$

Permissible LMC model

The model can be interpolated for other periods and still remains valid.

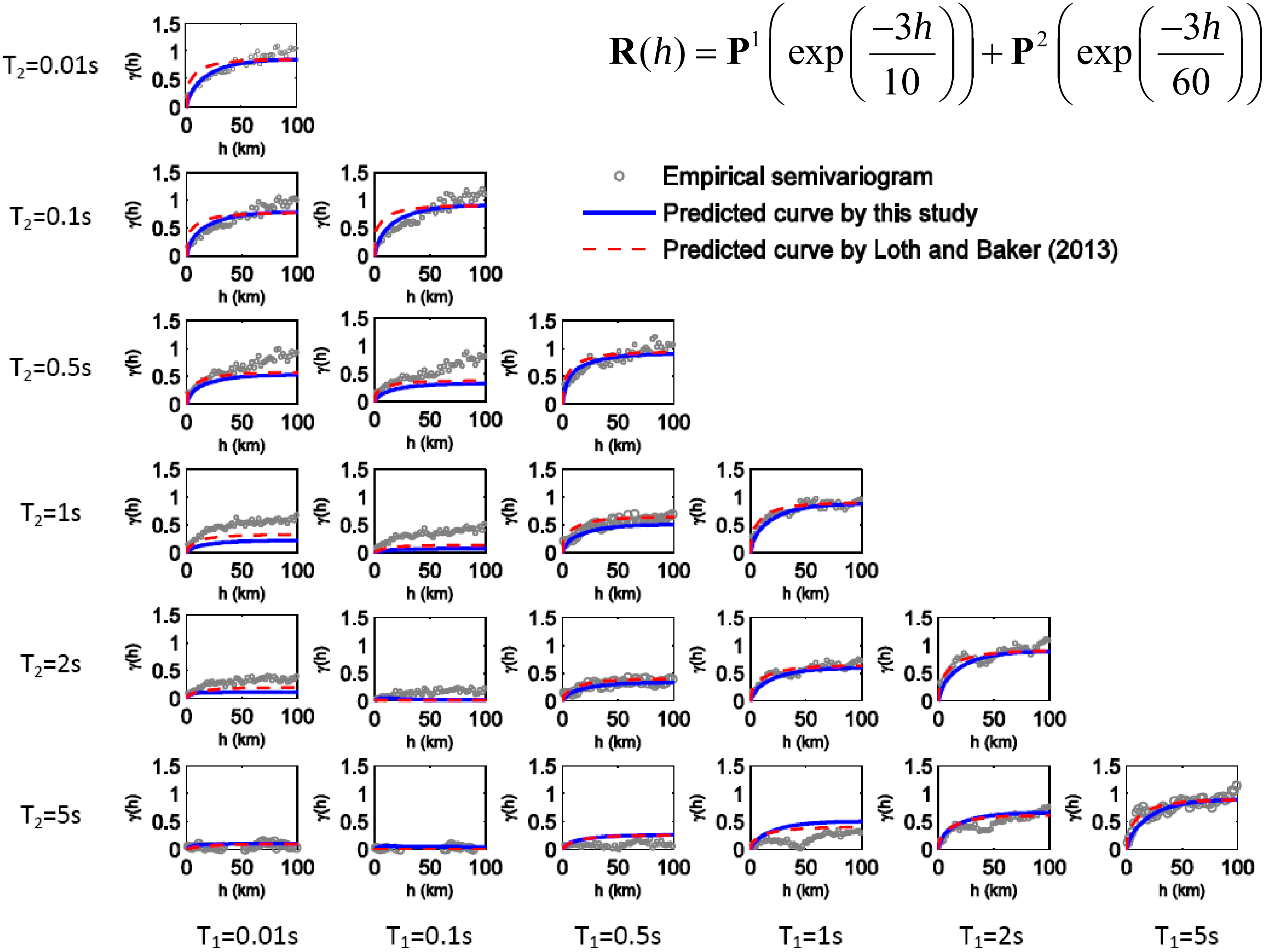
Model Prediction – Northridge Earthquake

$$\mathbf{R}(h) = \mathbf{P}^1\left(\exp\left(\frac{-3h}{10}\right)\right) + \mathbf{P}^2\left(\exp\left(\frac{-3h}{60}\right)\right)$$



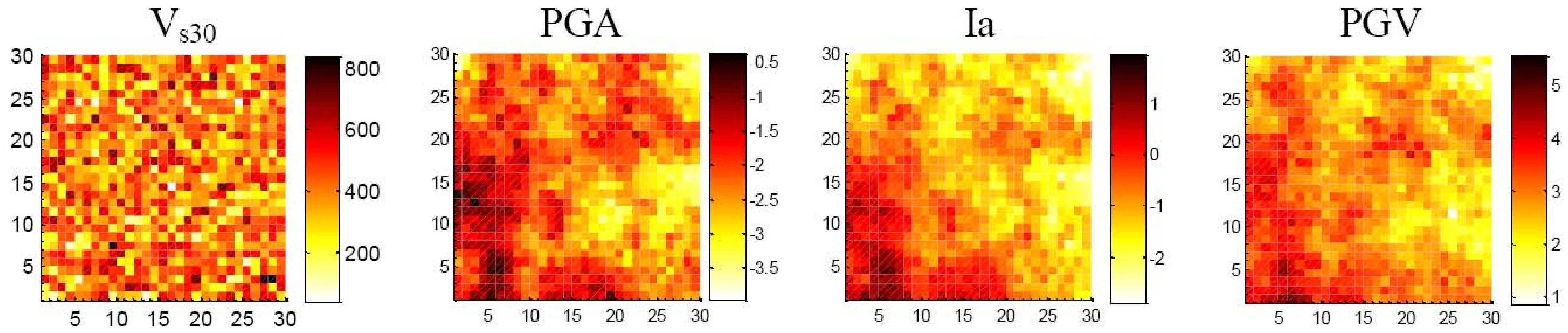
Model Prediction – Chi-Chi Earthquake

$$\mathbf{R}(h)=\mathbf{P}^{1}\left(\exp\left(\frac{-3h}{10}\right)\right)+\mathbf{P}^{2}\left(\exp\left(\frac{-3h}{60}\right)\right)$$

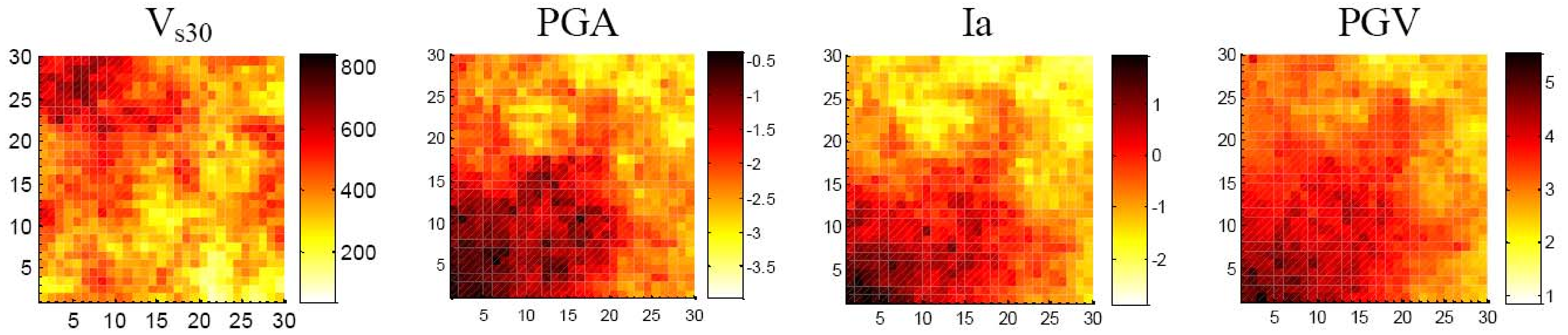


Model Applications -- Random Vector IM Fields

$R_{Vs30} = 0$ km

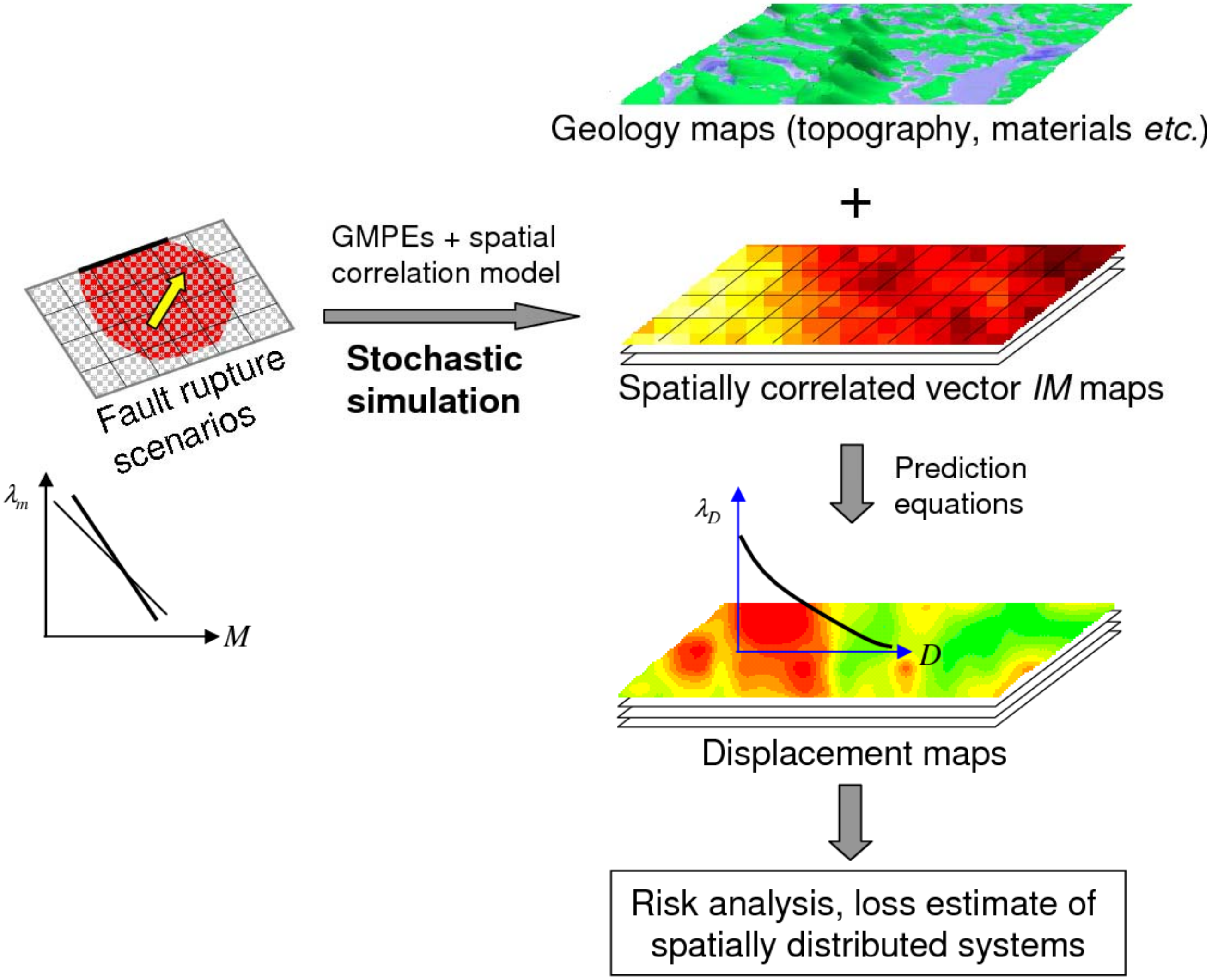

V_{s30} PGA Ia PGV

$R_{Vs30} = 25$ km


V_{s30} PGA Ia PGV

(PGA, Ia and PGV are in the natural log scale, in the unit of g, m/s and cm/s, respectively; V_{s30} is in the unit of m/s)

Model Applications -- Fully Probabilistic Approach using Spatially-correlated Vector IMs



cf. Du and Wang, Fully Probabilistic Seismic Displacement Analysis of Spatially Distributed Slopes Using Spatially Correlated Vector Intensity Measures, 2013 (under review)

Conclusions

- **Simple permissible spatial correlation models are developed for vector IMs (PGA, Ia, PGV, and Sa) using eleven recent earthquakes.**

- **The correlation range of V_{s30}, R_{Vs30}, is found to be a good indicator to characterize the regional geological conditions. In general, the spatial correlations of IMs becomes stronger for a homogeneous regional site condition.**

- **The spatial correlation models can be conveniently used in regional-specific seismic hazard analysis and loss estimation of spatially-distributed infrastructure.**

Questions?