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NUMERICAL ANALYSIS OF PILES IN ELASTO-PLASTIC SOILS UNDER AXIAL LOADING

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ABSTRACT

A finite element model has been developed to simulate nonlinear response of piles/drilled piers under axial loading. The nonlinear stress-strain behaviors of soils are modeled via Drucker-Prager and von Mises type plasticity. A parametric study is carried out to address the influence of various factors, such as soil friction approximation, dilatancy, effect of interface element and shear strength profile etc. on the prediction of pile behavior in elasto-plastic soils and key issues in the simulation are critically reviewed.

Keywords: Soil-pile interaction, Axial loaded pile, Drucker-Prager, von Mises plasticity

INTRODUCTION

Soil-structure interaction is an important topic in the development of a performance based design procedure. With the rapid advances of computing technology, finite element analysis is assuming more important role in engineering practice. The advantage of finite element analysis lies in its ability to accommodate complex soil stratigraphy and its potential for solving three-dimensional soil-structure interaction problems. However, to be successfully used in practical design, the soil model should be simple and can be easily calibrated by conventional field or lab testing. On the other hand, the model should be able to realistically capture the most important aspects of soil-structure nonlinearities.

Since it was first introduced, Drucker-Prager type model (Drucker and Prager, 1952) has been successfully adopted in analysis of geomaterials due to its relative simplicity. For example, a comprehensive nonlinear finite element analysis of vertically loaded pile was carried out using ABAQUS ™ (Trochanis et al. 1991). In this study, the surrounding soil was modeled using extended Drucker-Prager plasticity while the piles were modeled as linearly elastic material. Yang and Jeremić (2002) used non-associative Drucker-Prager for cohesionless soil and von Mises criterion for cohesive soil, and developed p-y curves for laterally loaded piles in multi-layered

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soil profiles. Although these previous analyses dealt with pile-soil interaction with various degrees of success, detailed information on model assumptions and uncertainties associated with model selection are not available. In the following sections, we describe a parametric study of the various factors associated with simple model simulation.

MODEL FORMULATION

Pressure-sensitive failure mechanism of soil is represented by a cone-shape yield surface:

$$F = \alpha I_1 + \sqrt{J_2} - Y = 0 \tag{1}$$

where $I_1 = \sigma_{ii}$ is the first invariant of stress tensor and $J_2 = \frac{1}{2} s_{ij} s_{ij}$ is the second invariant of deviatoric stress tensor $s_{ij} = \sigma_{ij} - \frac{1}{3} I_1 \delta_{ij}$. α and Y are material parameters related to the soil friction angle and the cohesion.

For small strain formulation, strain rate is usually additively decomposed into elastic and plastic components,

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{e} + \dot{\varepsilon}_{ij}^{p} \tag{2}$$

such that rate form of stress-strain relation can be written as

$$\dot{\sigma}_{ij} = C_{ijkl} (\dot{\varepsilon}_{kl} - \dot{\varepsilon}_{kl}^{p}) \tag{3}$$

To better describe volumetric behavior of soil, non-associative flow rule is usually adopted. The plastic flow is defined through potential surface Q with parameter β controlling soil dilatancy,

$$Q = \beta I_1 + \sqrt{J_2} - \tilde{Y} = 0 \tag{4}$$

Plastic strain rate is defined normal to the potential surface via,

$$\dot{\varepsilon}_{ij}^{p} = \lambda \frac{\partial Q}{\partial \sigma_{ij}} = \lambda \left(\frac{s_{ij}}{2\sqrt{J_2}} + \frac{1}{3} \beta \delta_{ij} \right) \tag{5}$$

where λ is plastic multiplier and can be determined by consistency condition.

Linear isotropic hardening rule was incorporated to describe hardening of yield surface through internal variable ξ,

$$Y = Y_0 + H\xi \tag{6}$$

where Y_0 and hardening modulus H are material parameters, and $\dot{\xi} = \lambda$

In our implementation, the return mapping algorithm (Simo & Taylor 1985, Simo & Hughes

1997) was employed to derive the algorithmic consistent tangent based on implicit backward Euler scheme to guarantee quadratic rate of global convergence. Note that for non-associative flow rule $\alpha \neq \beta$, major symmetry of the consistent tangent operator is lost. Further, cone apex is singular, and the normal to the potential surface is not defined. Special algorithmic treatment is needed around this region (Hofstetter and Taylor, 1991).

We implemented the Drucker-Prager model in OPENSEES --- Open System for Earthquake Engineering Simulation (http://opensees.berkeley.edu) hosted by Pacific Earthquake Engineering Research Center (PEER http://peer.berkeley.edu/), and the source code is available from the authors.

PILE RESPONSE IN COHESIONLESS SOIL

To demonstrate the capacity of Drucker-Prager model prediction, consider a 2.5 feet diameter circular concrete pier installed to 19 feet of depth in sand. The pile is vertically loaded on its top under drained condition. A finite element model was developed to simulate the pile behavior. Due to axisymmetry of this problem, only one half of the cross section is meshed using axisymmetric bilinear element. The mesh, shown in Fig. 1, extends to 40 feet in depth. The base of the mesh is fixed and only vertical movement is allowed along right hand side of the mesh and the axis of symmetry (the left hand side of the mesh). The pile is modeled with linearly elastic elements with a Young's modulus E_p= 20×10^6 kPa and Poisson's ratio v_p=0.1.

Important for ensuing capacity analysis, initial stress state of the soil should be properly simulated. Staged loading process was designed to enforce in-situ K_0 state of soil elements, where K_0 is the coefficient of earth pressure. The soil was initially assumed to be linearly elastic, with Poisson's ratio specified as v=K_0/(1+K_0). After vertically "consolidated" under its self weight to generate desired K_0 profile, the element materials were switched to behave elasto-plastically. In this simulation, soil parameters were chosen to be: submerged density ρ_s =1400 kg/m^3, Young's modulus E=10^5 kPa, Poisson's ratio v=0.3, K_0 =1-sinϕ where ϕ is the effective soil friction angle.

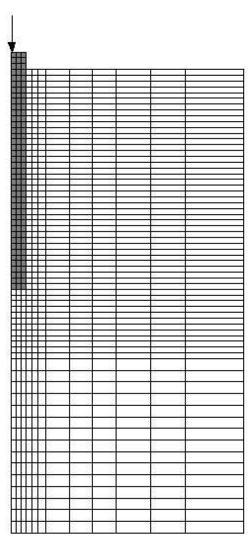


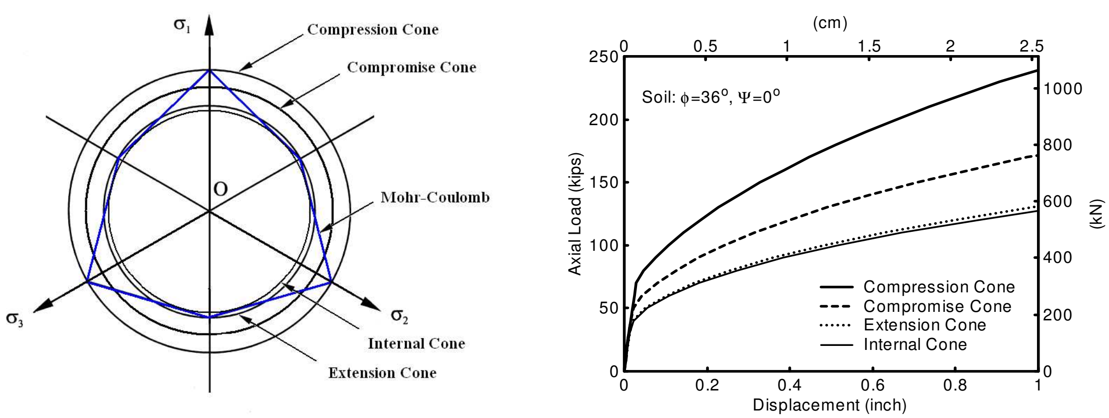
FIG. 1. FEM mesh

Sensible determination of model parameters with respect to physical properties of soil is one of the most important issues for a successful numerical simulation. To smoothly approximate the Mohr-Coulomb hexagon on the deviatoric stress plane (π plane), several strategies are available for determining Drucker-Prager cone parameters. As shown in Fig. 2(a), compression cone and extension cone are defined to match Mohr-Coulomb in either triaxial compression and triaxial extension. Internal cone is inscribed inside Mohr-Coulomb. A compromise cone can also be defined as kind of average between extension and compression approximations. For various Drucker-Prager approximations, Table 1 summarizes determination of model parameters with respect to soil friction angle ϕ, cohesion c and dilatancy angle Ψ .

TABLE 1. Model Parameter Determination

Drucker-Prager Approximation	Parameter α	Parameter β	Parameter Y
Compression Cone	$\alpha=\dfrac{2\sin\phi}{\sqrt{3}(3-\sin\phi)}$	$\beta=\dfrac{2\sin\psi}{\sqrt{3}(3-\sin\psi)}$	$Y=\dfrac{6c\cos\phi}{\sqrt{3}(3-\sin\phi)}$
Compromise Cone	$\alpha=\dfrac{2\sin\phi}{3\sqrt{3}}$	$\beta=\dfrac{2\sin\psi}{3\sqrt{3}}$	$Y=\dfrac{6c\cos\phi}{3\sqrt{3}}$
Extension Cone	$\alpha=\dfrac{2\sin\phi}{\sqrt{3}(3+\sin\phi)}$	$\beta=\dfrac{2\sin\psi}{\sqrt{3}(3+\sin\psi)}$	$Y=\dfrac{6c\cos\phi}{\sqrt{3}(3+\sin\phi)}$
Internal Cone	$\alpha=\dfrac{\sin\phi}{\sqrt{3}\left(3+\sin^2\phi\right)^{1/2}}$	$\beta=\dfrac{\sin\psi}{\sqrt{3}\left(3+\sin^2\psi\right)^{1/2}}$	$Y=\dfrac{3c\cos\phi}{\sqrt{3}\left(3+\sin^2\phi\right)^{1/2}}$

Pile head displacements vs. applied axial loads for various approximation schemes are shown in Fig 2(b), where soil parameters are ϕ=36°, c=0° and Ψ=0°. Surprisingly significant differences are found for various approximation schemes. The discrepancy lies in the fact that, the actual stress state of soil in loading (thus the actual mobilized friction angle) is considerably different than the cone approximation matching point. Similar observations were reported in Zienkiewicz et al. (1999) for footing loading and Schwiger (1994) for earth pressure simulation.



(a) Drucker-Prager approximations (b) Predicted pile response

FIG. 2. Effect of Drucker-Prager approximations

Effect of soil dilatancy was assessed by examining the load deflection response for varying dilatancy angle Ψ, as shown in Fig 3, where the soil friction angle ϕ=36°, cohesion c=0° and compression cone approximation were used in all cases. System response is shown very sensitive to the choice of Ψ. Pile capacity reached its yielding apparently in the non-dilatant soil (Ψ=0°), while the associative flow rule (Ψ=36°) predicted a nearly elastic response and was overly unconservative. The effect could be better appreciated from Eq. (5), which shows for any nonzero Ψ (and thus β), the Drucker-Prager model will predict continued plastic dilatancy in the persistent plastic loading without reaching a critical state. Similar findings and detailed discussions can be found in Potts (1999, 2001) for Mohr-Coulomb model.

Modeling of the interface behavior between soil and pile is important in the analysis of pile under vertical loading. In the above analyses, no specific interface element was used and the pile was assumed to be perfectly bounded with adjacent soils. To examine the effect of interface element, node-to-node zero-length frictional contact element (for example, see Wriggers 2002) was also developed in OPENSEES. The contact elements were placed along the shaft and the problem was re-analyzed. The friction angle of contact element ϕ' was chosen to be the same as soil friction angle ϕ for a clear comparison. In Fig. 4, pile responses with and without using interface elements are illustrated for $\phi = \phi' = 36°$, $c = 0°$ using compromise cone approximation. The significant difference as discussed above for full dilatant (Ψ=36°) and non-dilatant (Ψ=0°) soil is greatly suppressed by the presence of the interface contact elements. Instead of yielding through the Drucker-Prager type soil elements, the contact elements essentially enforce Mohr-Coulomb type failure mode along the shaft. So, with interface elements, lower capacity is always predicted. It is also noted that computational costs and numerical instability increase considerablely in simulations with interface elements.

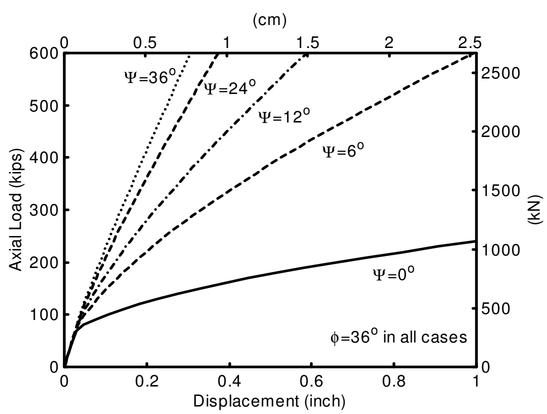


FIG. 3. Effect of dilatancy

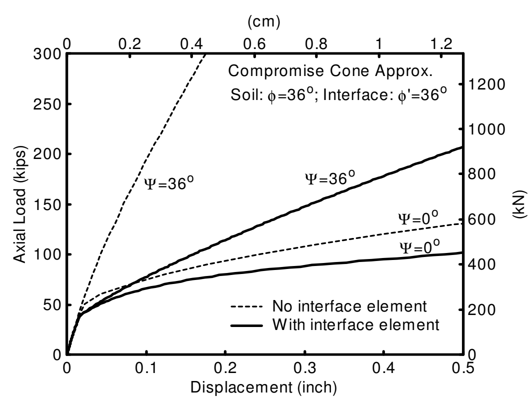


FIG. 4. Effect of interface element

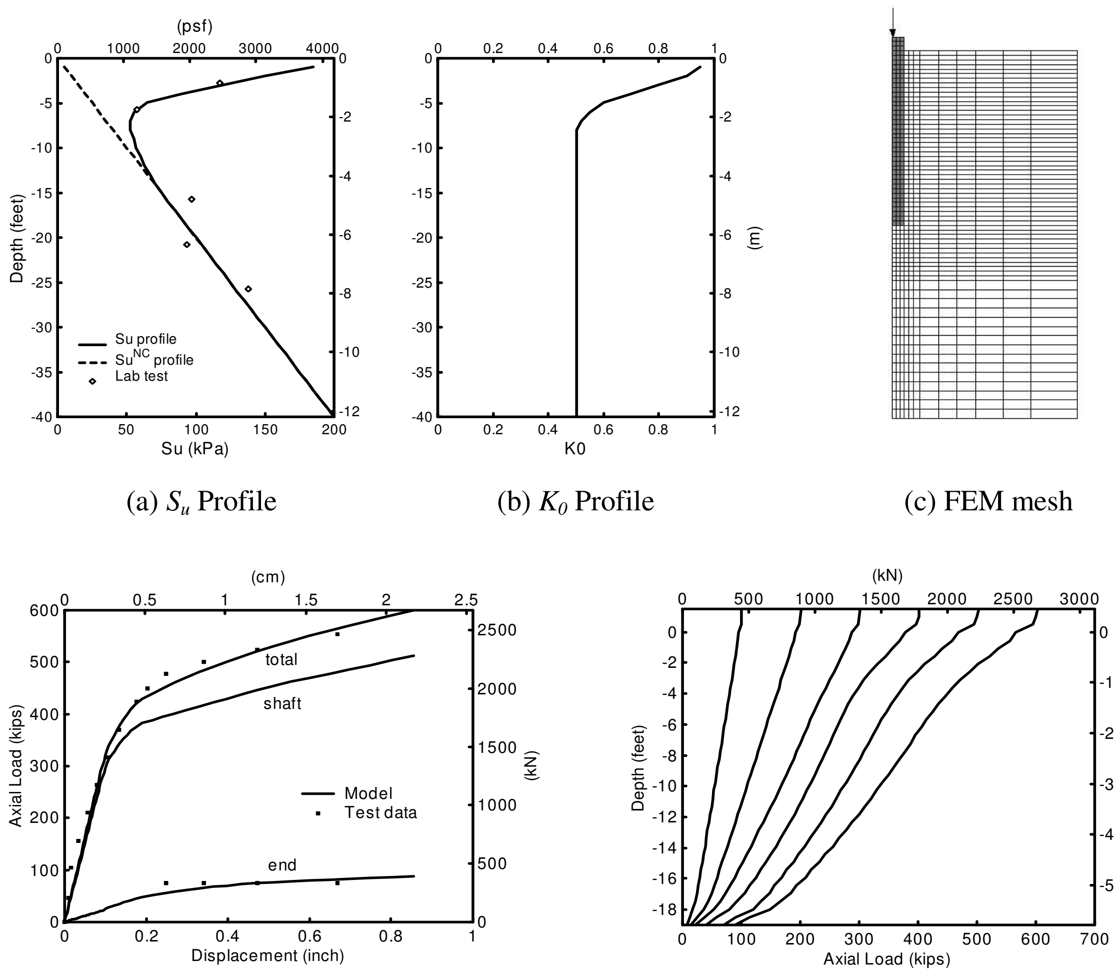
PILE RESPONSE IN COHESIVE SOIL

To further assess the validity of model, the numerical prediction was compared with a pier load test conducted recently near the University of California, Berkeley campus. The circular concrete pile was embedded to 19 feet depth, 2.5 feet in diameter and cast in place with downhole cleaning to ensure end bearing capacity. The site is mainly composed of hard to very stiff sandy clay, medium dense sandy silt and dense clayey sand. The undrained shear strength S_u profile estimated from unconfined compression test data and the estimated K_0 profile are shown in Figs. 5 (a)(b).

Drucker-Prager model can be reduced to von Mises type criterion by letting $\alpha=\beta=0$, which was used in total stress analysis of undrained response of cohesive soil. For this simulation, soil parameters are: total density ρ_t=2000 kg/m^3, Young's modulus E=10^5 kPa, Poisson's ratio v=0.49.

Strength parameter $Y_0 = \sqrt{4/3}\, S_u$ and K_0 were specified for each soil layer according to solid lines in Figs.5(a)(b). Slight hardening with modulus $H = 10^3$ kPa was also used. Again, the pile was modeled linearly elastic with a Young's modulus $E_p = 20\times10^6$ kPa and Poisson's ratio ν=0.1. The setup and FEM mesh (Fig 5(c)) are identical as used in previous analyses. Note that B-bar method was used in element formulation to avoid volumetric locking in the undrained analysis.

As shown in Fig. 5(d), we obtained excellent agreement between field measurements and model predictions, in view of both total capacity and end bearing components. Progressive failure mode of shear zone was also captured for the nonhomogenous soil strength profile. The load transfer curve along the pile length is shown in Fig. 5(e). As is well documented in field observation and literature, larger displacement is needed for end bearing capacity to be fully mobilized compared with shaft resistance.



(a) S_u Profile (b) K_0 Profile (c) FEM mesh

(d) Load-Displacement Curve (e) Load Transfer Curve

FIG. 5. Comparison of model prediction with field test

We re-analyzed the model for a homogenous soil strength profile to assess the influence of soil nonhomogeneity. Here, S_u = 84 kPa (averaged undrained shear strength over pile length plus one pile diameter) was assigned for all soil elements and K_0 = 0.5. As expected, soil strength is more likely to be mobilized simultaneously along the shaft in the uniform soil profile. As shown in Fig. 6, except for a small range of slightly brittle response, two profiles essentially predict the same load capacity. Apparently, for a stiff clay site, assumption of homogenous strength profile is sufficient for practical purpose, while care must be exercised for soft clay site.

Adequate account of the over consolidated (OC) soil strength is important especially for analysis of a short pier. The surface crust over top 8 feet is over consolidated with higher undrained shear strength that would be associated with a normally consolidated (NC) soil. If it is assumed the site is composed of all normally consolidated soil, and the shear strength is extrapolated in a conventional way, as plotted in dashed line in Fig. 5(a), to be

$$S_u^{NC} = 0.255\sigma_v' \tag{7}$$

where σ_v' is effective overburden stress. The predicted load capacity will be greatly underestimated, only up to 70% of actually capacity as shown in Fig. 7.

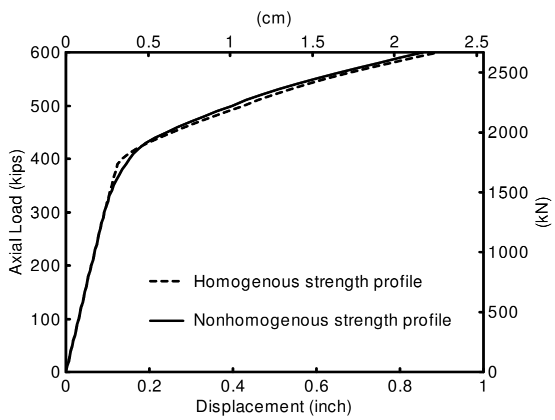


FIG. 6. Effect of soil strength profile

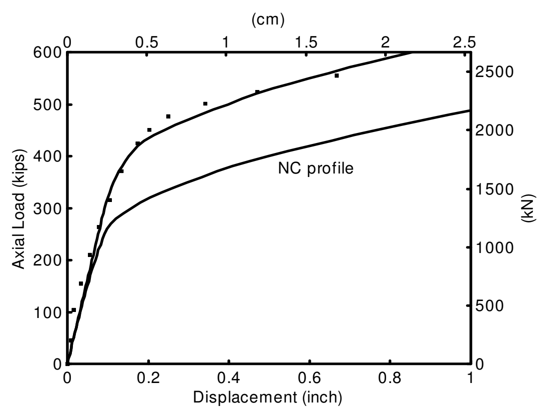


FIG. 7. Effect of OC strength

CONCLUSIONS

Drucker-Prager type plastic model has been developed to simulate vertically loaded pile response in cohesionless soil. Parametric study shows soil friction approximation scheme and dilatancy modeling impose great uncertainties in system response. Parameters should be assigned according to the actually mobilized friction angle and non-associative flow rule is preferred for this type of simple model. Uncertainties of soil behavior can be regulated by the presence of frictional contact elements.

Pile response in cohesive soil was also successfully simulated using von Mises type criterion and it is in excellent agreement with field test data. It is found that an accurate undrained shear strength profile is of the highest importance for the capacity analysis. Proper account of over consolidated crust is important especially for short pier capacity simulation, while nonhomogeneity of undrained shear strength distribution is not critical for the stiff clay site.

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