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# A Modified Frequency-Dependent Equivalent Linear Method for Seismic Site Response Analyses and Model Validation using KiK-Net Borehole Arrays

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**ABSTRACT**

A modified frequency-dependent equivalent linear method (M-FD-EQL) is proposed to improve 1D site response at high frequencies. A combination factor is used to interpolate the strain spectrum between the equivalent linear method (EQL) and the frequency-dependent equivalent linear method (FD-EQL). Extensive validation tests have been conducted to benchmark the proposed method with nonlinear analyses of generic sites, and 383 borehole array data from the KiK-net. Based on model validation, an optimal combination factor of 0.2–0.3 in the M-FD-EQL scheme gives an overall best result, and provides improved site response estimates at high frequencies compared with the EQL and FD-EQL methods.

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**KEYWORDS**
Site Response Analyses; Equivalent Linear Method; Borehole Array; Model Validation; High-Frequency Response

## 1. Introduction

In the past decades, one-dimensional seismic site response analyses have been widely used to quantify the effect of soil deposits on propagated seismic waves [Kramer, 1996]. The method assumes that both soils and bedrock are horizontally layered, which extend infinitely in the horizontal direction, and only vertically propagated horizontal shear waves are considered in the analysis. One of the major challenges in site response analysis is to properly approximate soil nonlinearity during cyclic loading. Over the years, different schemes have been developed to address this issue, including the well-known 1D equivalent linear method (EQL) that has been implemented into the widely used computer program SHAKE [Schnabel *et al.*, 1972; Idriss and Sun, 1992]. The EQL scheme utilizes an iterative procedure to prescribe elastic shear modulus and damping in each soil layer that are compatible with the induced strain level. However, the EQL method is still a linear method of analysis with some significant limitations. Due to the fact that EQL method utilizes strain-compatible yet constant soil properties throughout the entire analysis, the computed surface motions are often underestimated (overdamped) at high frequencies. This is because strains associated with high-frequency response are usually of small amplitudes. Correspondingly, the associated damping is substantially smaller than that used in the EQL analysis. This limitation has been recently reported in a couple of studies

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[Kaklamanos *et al.*, 2013; Zalachoris and Rathje, 2015], where the EQL analyses are compared with recorded ground motions from borehole arrays.

To address the above limitation, the frequency-dependent equivalent linear method (FD-EQL) has been developed, which utilizes a strain spectrum to estimate strain amplitudes associated with different frequencies [Kausel and Assimaki, 2002; Assimaki and Kausel, 2002]. For one-dimensional plane shear wave propagation in an unbounded medium, the shear strain is always proportional to the particle velocity via the following equation:

$$\gamma = \frac{\partial u(t - x/V_s)}{\partial x} = -\frac{\dot{u}}{V_s} \tag{1}$$

where $\gamma$ denotes the shear strain, $V_s$ is the shear wave velocity of the soil, and $\dot{u}$ is the particle velocity. In this case, the shear strain spectrum $\gamma(\omega)$ can be determined as Fourier spectrum of a velocity time history divided by the shear wave velocity [Kausel and Assimaki, 2002]. In practice, a smoothed piecewise function can be used to approximate the strain spectrum, which assumes a constant $\gamma_0$ at frequencies smaller than $\omega_0$, and an exponential function at frequencies greater than $\omega_0$ as follows:

$$\left|\frac{\gamma(\omega)}{\gamma_0}\right| = \begin{cases} 1, & \omega \le \omega_0 \\ \dfrac{\exp\left[-\alpha\left(\frac{\omega}{\omega_0}-1\right)\right]}{\left(\frac{\omega}{\omega_0}\right)^{\beta}}, & \omega > \omega_0 \end{cases} \tag{2}$$

where $\omega_0 = \int_0^\infty \omega\gamma(\omega)d\omega / \int_0^\infty \gamma(\omega)d\omega$ is the mean frequency, $\gamma_0 = \frac{1}{\omega_0}\int_0^{\omega_0} \gamma(\omega)d\omega$ is the strain averaged over a frequency range from 0 to $\omega_0$, $\alpha$ and $\beta$ are two fitting parameters to control the shape of the smoothed strain spectrum. Figure 1a shows an example of the strain spectrum with a smoothed fitting function via Eq. (2). Using the strain spectrum $\gamma(\omega)$, the conventional modulus reduction and damping curve $G(\gamma)/G_{\max}$ and $\xi(\gamma)$ can be readily converted to frequency-dependent modulus and damping $G(\omega)/G_{\max}$ and $\xi(\omega)$, which is schematically illustrated in Fig. 1b.

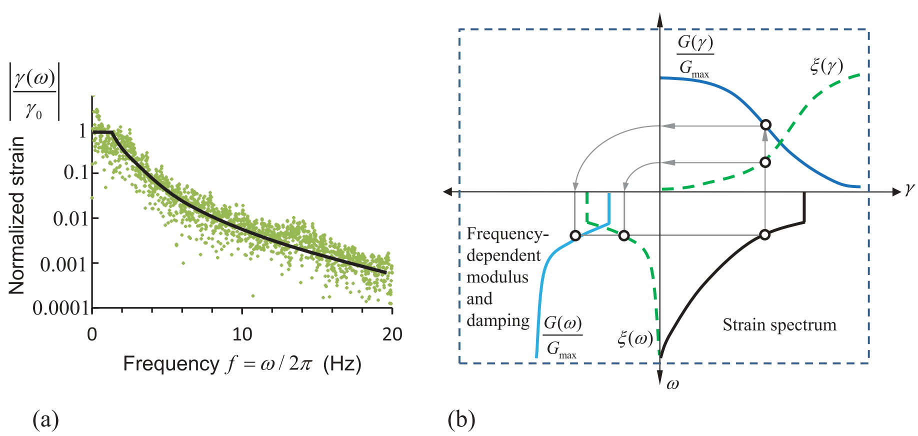



**Figure 1.** (a) An example of strain spectrum ($\alpha$ = 0.12 and $\beta$ = 2.2, dots are discrete data of the strain spectrum, the solid line is a smoothed fitting function); (b) schematic of the evaluation of frequency-dependent material parameters [adapted from Kausel and Assimaki, 2002].

Figure 2 illustrates the flow chart of the FD-EQL method. Similar to the EQL method, the solution scheme is conducted in the frequency domain. First, the irregular input motion is decomposed into harmonic components at different frequencies by Fourier transform. For each frequency $\omega_i$, the frequency-dependent soil modulus and damping $(G(\omega_i),\xi(\omega_i))$ are assigned according to the strain spectrum. Therefore, a large stiffness and a small damping ratio are associated with high-frequency excitation. Summation of system responses under harmonic motions produces time history solutions in soil layers. The strain spectrum and frequency-dependent modulus and damping will be updated from the time history solution. Iterations are carried out until the strain solutions are converged in each soil layer.

Yoshida *et al.* [2002] demonstrated that the FD-EQL method can overcome the limitation of the EQL method as mentioned before. However, it should be clarified that although material parameters of soils are assumed to be frequency-dependent during the FD-EQL analysis, the frequency of loading actually has negligible effects on soil modulus and damping over the frequency range of interest for most earthquakes (0.1–30 Hz) [Sun *et al.*, 1988].

Yet, it has been reported that the FD-EQL method may overestimate ground response at high frequencies [Kwak *et al.*, 2008]. Zalachoris and Rathje [2015] compared results of site responses with borehole observations, and found that FD-EQL analyses overpredict site amplification by as much as 75% at peak strain larger than 0.1% and at periods less than 0.4 s. Here, we try to explain the phenomenon from a physics point of view: as the frequency-domain solution is a summation of system response under harmonic motions at different frequencies, the principle of superposition is only valid when the system is

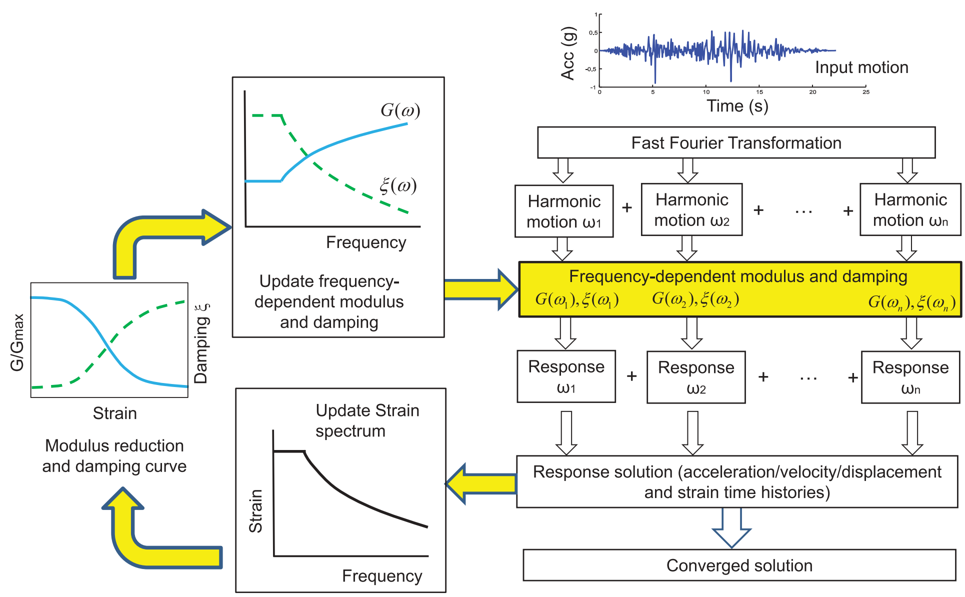



**Figure 2.** A flow chart of the frequency-dependent equivalent linear (FD-EQL) site response computation techniques.

identical at different frequencies, which can be well applied to the conventional EQL approach because constant soil properties are adopted in the analysis. Yet, the FD-EQL method assigns different soil properties at different frequencies; direct superposition may result in unrealistic ground responses at high frequencies, because complicated interaction between system responses at different frequencies is not considered in the FD-EQL method. If this interaction is considered, high-frequency loading/unloading response cycles should be less frequent than that implied by direct superposition. The overprediction effects can be significant for strong motions or motions with rich high-frequency contents.

In this study, a modified frequency-dependent equivalent linear method (M-FD-EQL) is proposed to improve 1D site response using the equivalent linear method. A combination factor $f$ is proposed to improve simulation at high frequencies by interpolating the response solution between the EQL and FD-EQL. Several generic sites are used to calibrate this combination factor against fully nonlinear analyses (NL). In addition, the M-FD-EQL method is validated using recordings from the KiK-net borehole arrays in Japan. Simultaneous surface and downhole recordings from the downhole array allow for direct monitoring of the ground amplification when waves travel to the surface, which provides a unique opportunity to validate the numerical simulation against instrumented data.

## 2. Modified Frequency-Dependent Equivalent Linear Method (M-FD-EQL)

### 2.1. *Modified Frequency-Dependent Equivalent Linear Method*

Figure 2 presents schematic representation of the FD-EQL method. A salient feature of this method is to use strain spectrum to assign frequency-dependent soil properties. The framework can also be applied to the EQL method, except that a constant "effective strain" is used for all frequencies. In other words, the strain spectrum for the EQL method can be regarded as a constant line, which can be determined as 65% of the maximum strain $\gamma_{EQL}(\omega) = 0.65\gamma_{\max}$. Figure 3 shows the strain spectra for the EQL, FD-EQL, and M-FD-EQL methods. By

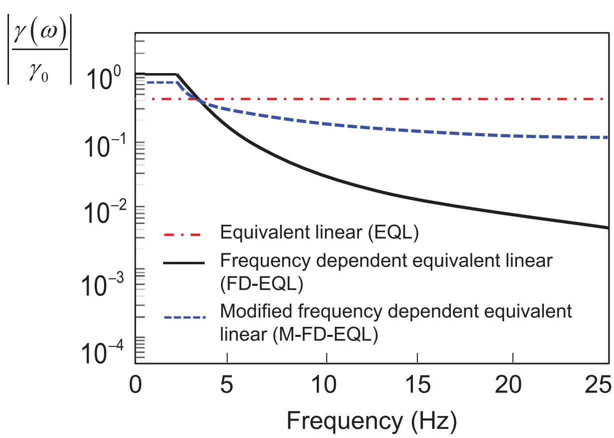



**Figure 3.** Strain spectra for the equivalent linear (EQL), frequency-dependent equivalent linear (FD-EQL) and modified frequency-dependent equivalent linear methods (M-FD-EQL).

incorporating the $\gamma_{EQL}(\omega)$and $\gamma_{FD-EQL}(\omega)$ in a natural log scale, the strain spectrum for M-FD-EQL method can be obtained following Eq. (3) and Eq. (4):

$$\ln \gamma_{M-FD-EQL}(\omega) = (1 - factor) \times \ln \gamma_{EQL}(\omega) + factor \times \ln \gamma_{FD-EQL}(\omega) \tag{3}$$

Alternatively,

$$\gamma_{M-FD-EQL}(\omega) = \gamma_{EQL}^{1-factor}(\omega) \cdot \gamma_{FD-EQL}^{factor}(\omega) \tag{4}$$

where *factor* denotes a combination factor ranging from 0 to 1. Apparently, *factor* = 0 represents the EQL method, and *factor* = 1 reduces to the FD-EQL method.

Besides equivalent linear analysis, fully nonlinear time-domain analysis can capture the complicated soil behavior of under earthquake loading [e.g., Wang and Xie, 2014; Ye and Wang, 2015; 2016; Ye *et al.*, 2016]. Previous researchers reported that NL simulate site response more accurately, especially when the induced strain is larger than 0.4% [Kaklamanos *et al.*, 2013]. Among many nonlinear models, DEEPSOIL is one of the most widely used for nonlinear 1D site response analyses [Hashash and Park, 2002; Park and Hashash, 2004; Stewart and Kwok, 2008], where stress-strain relation of the soil is fully nonlinear following prescribed modulus reduction and damping curve. Comparisons of DEEPSOIL and the EQL method have been extensively conducted by many researchers [Carlton and Tokimatsu, 2016; Kim *et al.*, 2016; Eskandarinejad *et al.*, 2017].

To demonstrate the influence of the combination factor in the M-FD-EQL method, seismic site responses were conducted using a generic site (NEHRP site class C, averaged shear wave velocity in upper 30 m $V_{s30}$ = 460 m/s). Results of the M-FD-EQL method using different combination factors are compared with those obtained from NL using DEEPSOIL. Figure 4a shows the input motion recorded at the Cerro Prieto station during

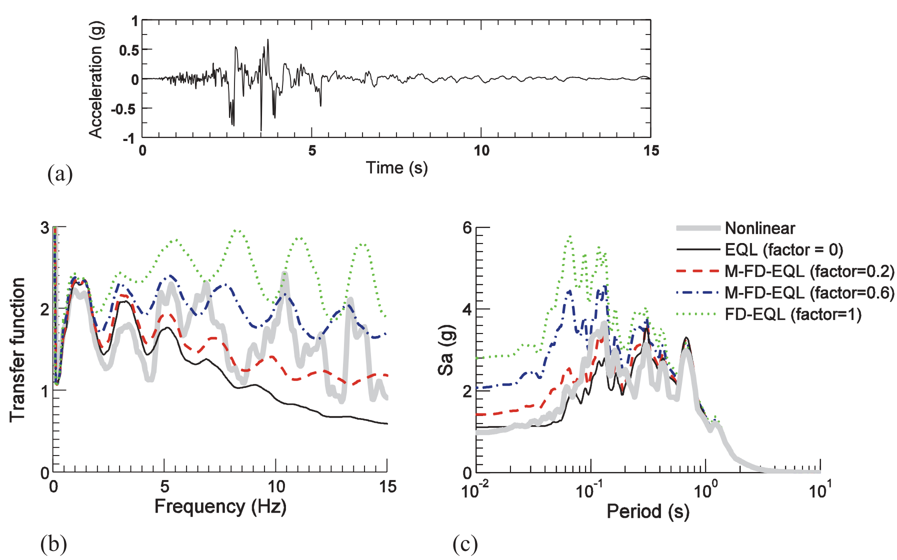



**Figure 4.** (a) Acceleration time history recorded at the Cerro Prieto station during the 1987 **M**5.5 Baja California earthquake as an input motion (b) Spectral accelerations and (c) transfer functions of ground response of a NEHRP class C site by the modified M-FD-EQL method (black line) and a fully nonlinear method (gray line) using the input motion.

the 1987 M5.5 Baja California earthquake with peak ground acceleration (PGA) of 0.89 g. The shear wave velocity profile for NEHRP site class C is shown in Fig. 5, and soil modulus reduction and damping curves follow Darendeli [2001].

Figure 4b,c compares the transfer functions and spectral accelerations (Sa's) of the ground response by varying the M-FD-EQL combination factor from 0 to 0.2, 0.6, and 1. It can be observed that the EQL method (*factor* = 0) has the lowest transfer function for frequencies over 5 Hz compared with other methods, implying that the high-frequency amplification is underestimated. The transfer function increases at high frequencies as the combination factor increases. Correspondingly, the ordinates of Sa's increase at the high-frequency range when the combination factor increases. Note that the nonlinear analyses result in a ground PGA of around 1 g, while the FD-EQL method (factor = 1) produces a PGA as high as 2.8 g, implying that the high-frequency content might be overly amplified using the FD-EQL method.

## 2.2. Influence of Combination Factors for Site-Specific Analyses

The influence of the combination factor is systematically studied using three generic sites NEHRP site classes C, D, and E [FEMA, 2003], with the shear wave velocity profiles of these sites illustrated in Fig. 5. The soil modulus reduction and damping curves were assigned following the Darendeli [2001] model.

The PEER-NGA database is used to study the performance of the EQL, FD-EQL, M-FD-EQL methods, and the results are compared with those obtained using the NL method (DEEPSOIL). Note that many previous studies [e.g., Kaklamanos *et al.*, 2015; Zalachoris and Rathje, 2015] have shown that one-dimensional NL analyses have considerable uncertainty, and they may not represent "true" or observed ground responses. The comparison to the NL method conducted herein is helpful for

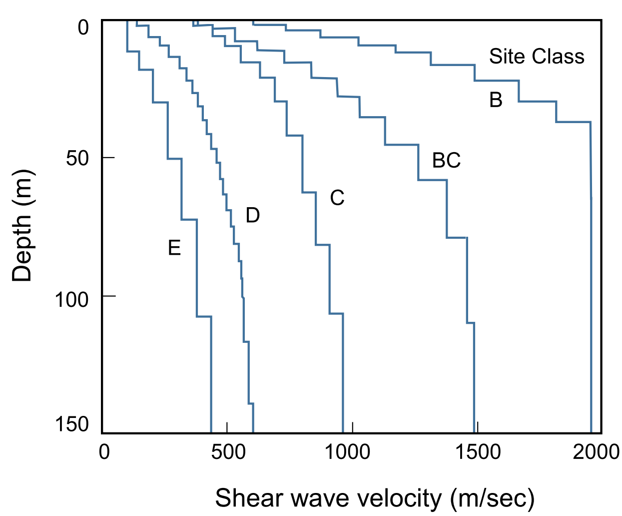



**Figure 5.** Average shear wave velocity profiles for generic NEHRP sites [adapted from Hartzell *et al.*, 2004].

understanding the behavior of the models with respect to different combination factors. Determination of the optimal combination factor will be verified using downhole array data in Section 3.

The PEER-NGA database contains a total of 173 earthquakes from California, Japan, Taiwan and other seismic active regions, with a total of 3551 three-directional acceleration time histories [Chiou *et al*., 2008]. Input motions are selected based on the following criteria: (a) PGA larger than 0.03 g, because these motions can induce intense ground shaking and soil nonlinearity at a site; (b) stiff soil or rock sites ($V_{s30}$ greater than 400 m/s); (c) ground motions are recorded at free field. By applying the above criteria, a total of 996 motions have been selected and are used as input motions in site response analyses.

To compare the site responses, a residual term $r_{\ln Sa}$ is defined to quantify the relative difference between Sa's (in a natural log scale) computed using the NL method (DEEPSOIL) and the M-FD-EQL method via Eq. (5):

$$r_{\ln Sa} = \ln(Sa_{NL}(T)) - \ln(Sa_{M-FD-EQL}(T)) \tag{5}$$

where $Sa_{NL}(T)$denotes the spectral acceleration at period $T$ computed using the non-linear method, $Sa_{M-FD-EQL}(T)$ represents the spectral acceleration at period $T$ computed using the modified frequency-dependent equivalent linear method. The mean and standard deviation of residuals for a variety of combination factors are demonstrated as contour plots against the vibration period $T$ and surface PGA in Figs. 6, 7, and 8 for NEHRP site classes C, D, and E, respectively.

Figure 6 shows the results for NEHRP site class C. On average, the mean of residuals falls in the range of −0.4 to 0.15 for EQL, FD-EQL, and M-FD-EQL methods, while the standard deviation is in the range of 0–0.2. It is observed that for the EQL method, the mean of residuals is approximately 0.1 for large PGAs at the spectral period of 0.1 s. The slight underprediction is due to overdamping of high frequencies at large strain levels. For the FD-EQL method, the mean of residuals is generally negative (overestimation) for all the period range. The over-prediction is more pronounced for PGA >0.2 g and $T$ < 0.1 s due to the fact that small damping is adopted at high frequencies by the FD-EQL method.

The EQL method also underestimates large motions at short periods for site class D, as shown in Fig. 7. The means of residuals are positive at PGA >0.05 g and $T$ = 0.08–0.2 s. The FD-EQL method significantly overpredicts the response, as the mean of residuals smaller than −0.4 for $T$ < 1 s and PGA >0.1 g. On the other hand, the absolute values of mean of residuals using the M-FD-EQL method are smaller than 0.2 at most periods. The hypotheses of the optimal combination factor will be verified using downhole array data in the next section. Similar ground response analyses have been conducted for NEHRP site class E, as shown in Fig. 8. Both of the EQL and M-FD-EQL methods (with a combination factor of 0.2) provide relatively small absolute values of mean of residuals (smaller than 0.3) at short periods. On the other hand, the FD-EQL again yields overestimation of ground responses, as the means of residuals are generally smaller than −0.4 (overestimation) for $T$ < 1 s and PGA >0.1 g.

Results from Figs. 6–8 all show that the M-FD-EQL method with a combination factor of 0.2 can effectively reduce the biases associated with the EQL method. The greatest improvement is in the range of 0.1 s and there is less improvement for shorter spectral

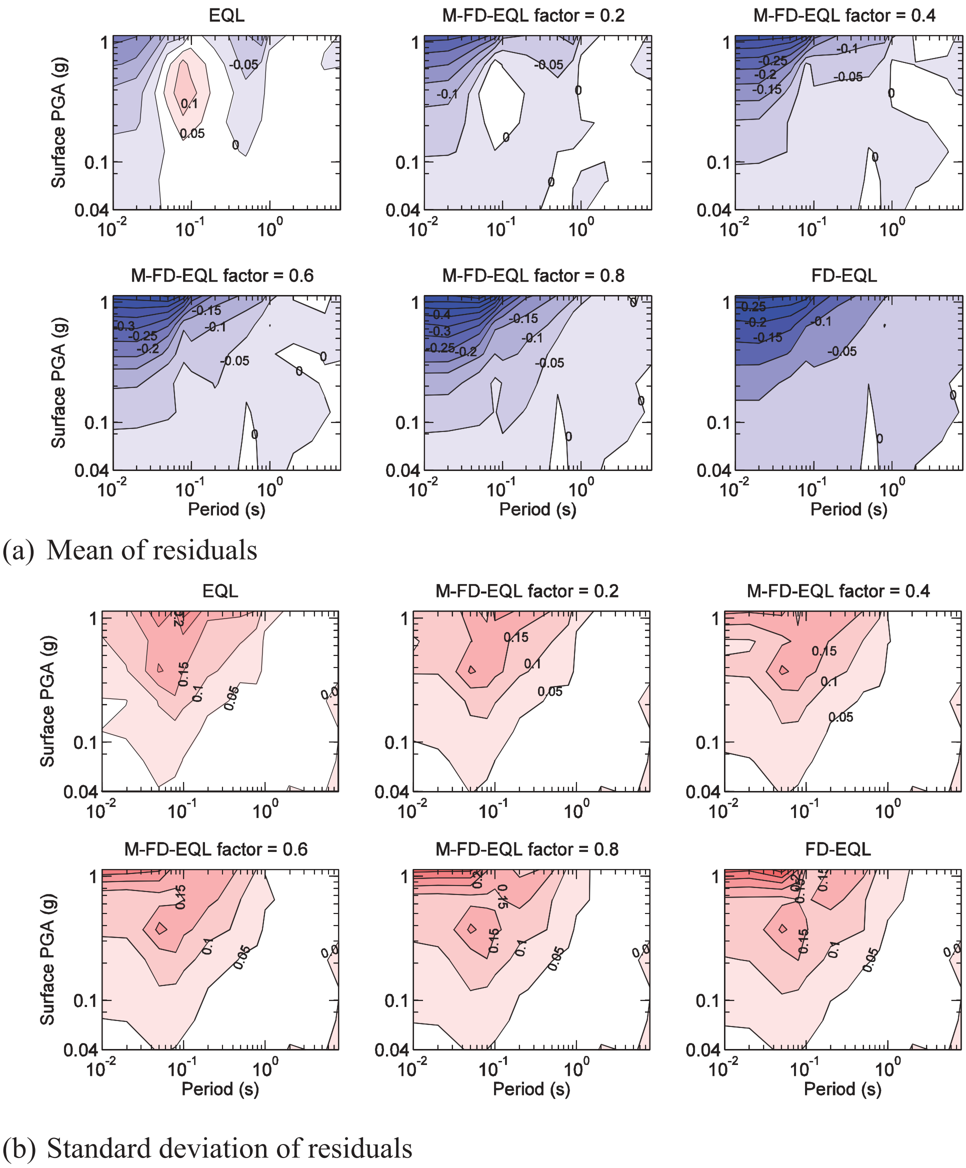


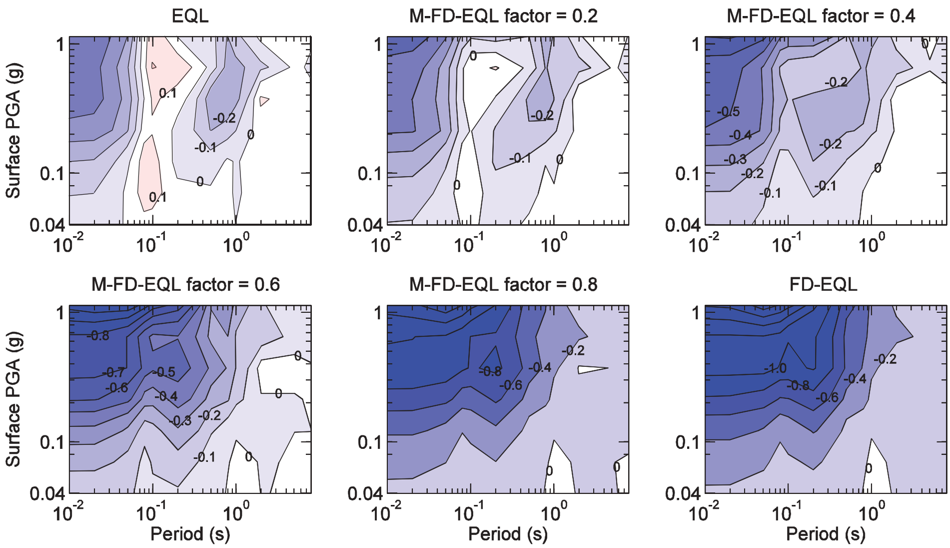

**Figure 6.** Contours of (a) mean and (b) standard deviation of residuals on NEHRP site class C.

periods. For very large PGAs, NL analyses would be preferred over various equivalent linear analyses anyway.

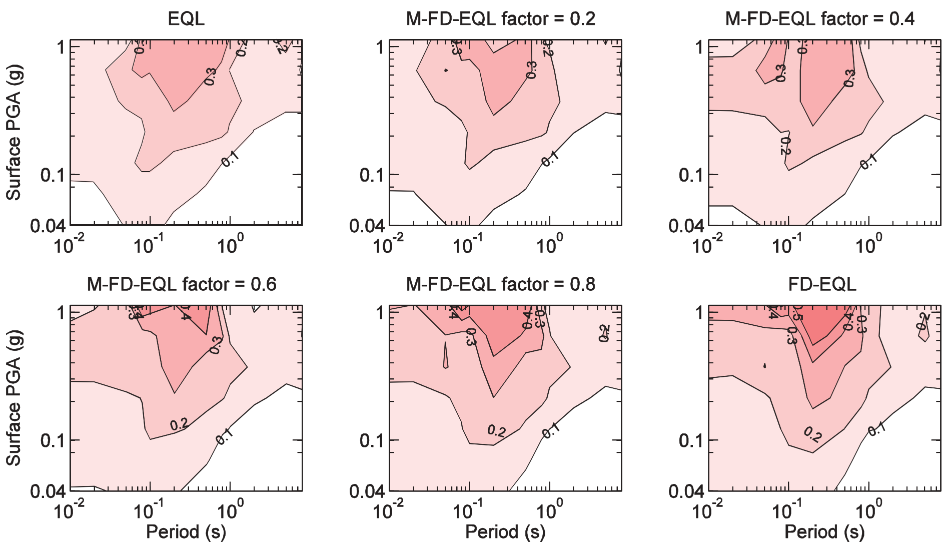
## 3. Validation of the M-FD-EQL Method Using KiK-net Downhole Array Data

### 3.1. *Subset of KiK-net Downhole Array Data*

In this study, the accuracy of one-dimensional EQL, FD-EQL, and M-FD-EQL site response methods is validated by comparing model predictions against recordings



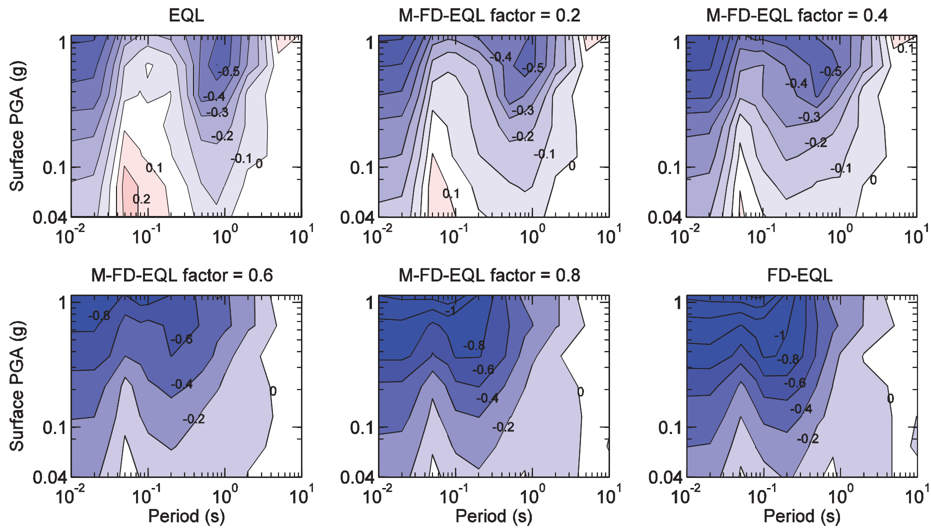

(a) Mean of residuals



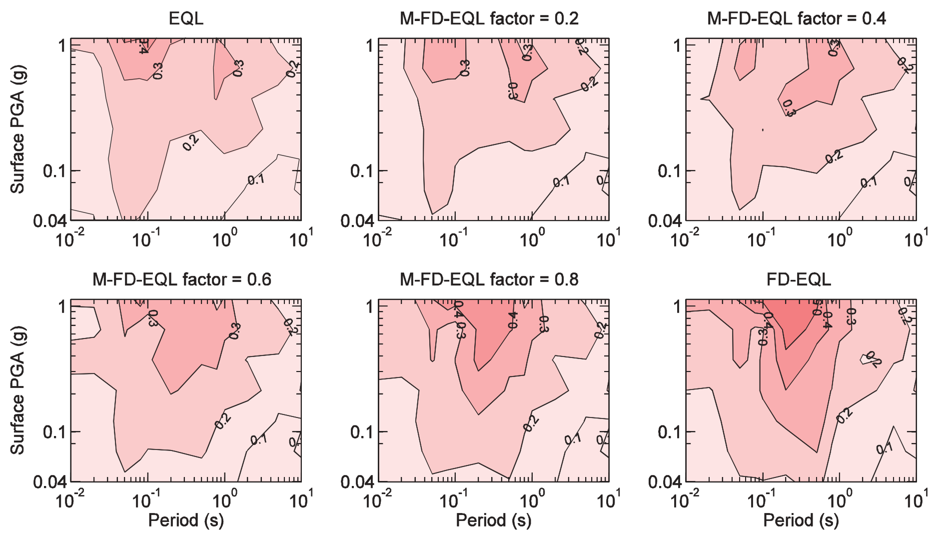

(b) Standard deviation of residuals

**Figure 7.** Contours of (a) mean and (b) standard deviation of residuals on NEHRP site class D.

from KiK-net borehole arrays. The borehole network consists of 689 stations with shear- and compressive-wave velocity profiles of subsurface geology units to the bedrock. Each KiK-Net station is equipped with three directional accelerometers at both ground surface and the bottom of the borehole. Simultaneous surface and downhole recordings from the downhole array allow direct monitoring of the ground amplification when waves travel to the surface. However, previous researchers identified that one-dimensional assumption does not hold for some of stations due to topography effects and spatial heterogeneity of site conditions [Thompson *et al.*,




(a) Mean of residuals




(b) Standard deviation of residuals

**Figure 8.** Contours of (a) mean and (b) standard deviation of residuals on NEHRP site class E.

2012]. To select the most appropriate stations for calibration and validation of one dimensional site response analyses, Thompson *et al.* [2012] proposed a classification scheme and selected 100 stations based on the following criteria: (a) the selected station should have at least one recording with PGA >0.3 g, and (b) at least ten recordings with PGA <0.1 g. The first requirement ensures that the selected stations have strong motions in order to study the highly nonlinear ground response. The second requirement ensures that there is sufficient number of weak motions included

**Table 1.** Summary of the selected 16 KiK-net stations in this study.

| Station | Latitude | Longitude | Vs30 (m/s) | NEHRP site classification | Number of recorded events |
|---|---|---|---|---|---|
| FKSH11 | 37.1976 | 140.3420 | 240 | D | 15 |
| FKSH14 | 37.0233 | 140.9736 | 237 | D | 19 |
| IBRH10 | 36.1078 | 139.9919 | 144 | E | 11 |
| IBRH13 | 36.7924 | 140.5784 | 335 | D | 95 |
| IBRH17 | 36.0822 | 140.3171 | 301 | D | 21 |
| IWTH02 | 39.8222 | 141.3861 | 390 | C | 54 |
| IWTH08 | 40.2658 | 141.7867 | 305 | D | 14 |
| IWTH24 | 37.5382 | 138.6174 | 486 | C | 15 |
| IWTH27 | 39.0278 | 141.5356 | 670 | C | 43 |
| KSRH06 | 43.2175 | 144.4325 | 326 | D | 12 |
| KSRH07 | 43.1333 | 144.3314 | 204 | D | 11 |
| KSRH10 | 43.2058 | 145.1208 | 213 | D | 20 |
| NIGH11 | 37.1697 | 138.7472 | 375 | C | 17 |
| NMRH04 | 43.3953 | 145.1264 | 168 | E | 8 |
| TCGH12 | 36.6928 | 139.9875 | 344 | D | 7 |
| TKCH08 | 42.4839 | 143.1558 | 353 | D | 21 |

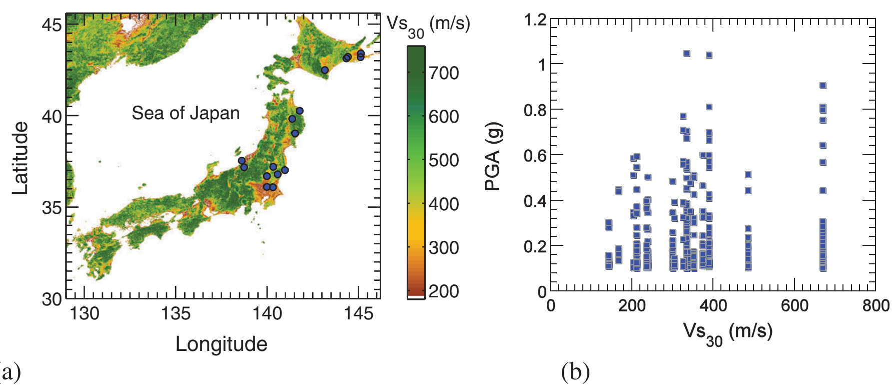



**Figure 9.** (a) Location of 16 strong-motion stations adopted in this study; (b) distribution of surface PGA versus $V_{s30}$ for the 383 selected records used in this study.

in the analysis in order to examine the effectiveness of one-dimensional assumption without being influenced by nonlinear soil behaviors. Of the 100 stations, 16 stations are further identified as their transfer functions have low inter-event variability and good fitness to the transfer functions computed by one-dimensional ground response analysis. Therefore, these 16 stations are adopted in the current study to validate the M-FD-EQL approach against observed data.

As summarized in Table 1, the selected 16 stations cover NEHRP site class C, D, and E. A total of 383 sets of array data are recorded during the past earthquakes. Figure 9 shows the location of 16 stations and the distribution of PGA versus $V_{s30}$ of the records. These sites were modeled using the documented *in situ* shear wave velocity profiles, and soil density is estimated based on the P-wave velocity following Boore [2016]. Nonlinear soil modulus reduction and damping curves for each unit are selected based on Darendeli [2001], according to the effective stress and plasticity index estimated from soil types.

### 3.2. Model Validation

Site responses are numerically computed using the EQL, FD-EQL, and M-FD-EQL methods and compared with real observation in the KiK-net database. To quantify the difference between the computed and recorded motions, a residual term *r* is defined in

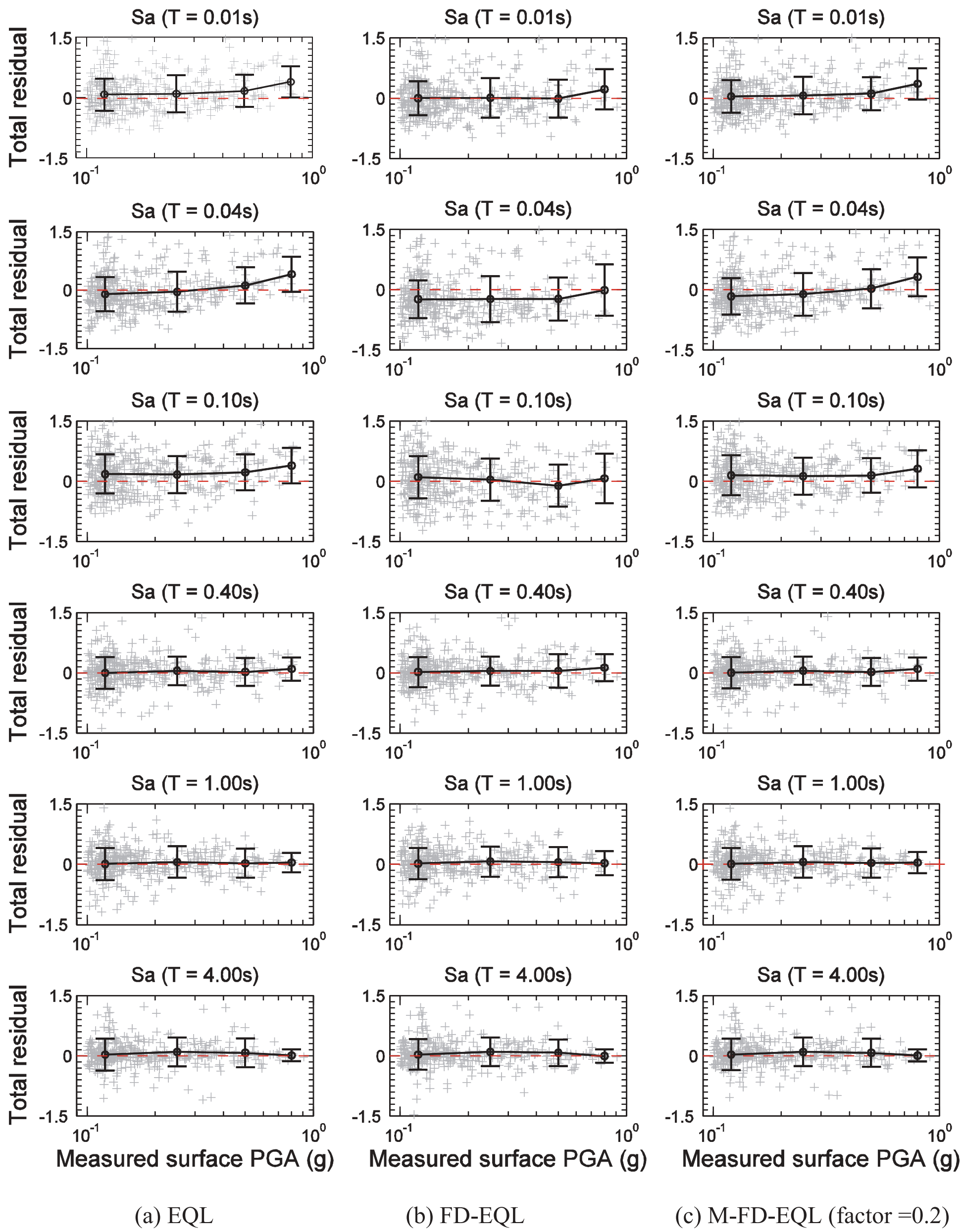



**Figure 10.** Total residual of ground responses for different spectra periods using the (a) EQL method, (b) FD-EQL method, and (c) M-FD-EQL method (factor = 0.2).

terms of spectral accelerations, $Sa(T)$, at period $T$ in a natural log scale as the following Eq. (6):

$$r(T) = \ln\left(Sa(T)_{Recorded}\right) - \ln\left(Sa(T)_{Computed}\right) \tag{6}$$

where a positive residual represents underprediction by the model, while a negative residual denotes overprediction. Figure 10 presents the residuals using a total of 383 records from the KiK-net database using the EQL, the FD-EQL, and the M-FD-EQL methods. It can be observed from Fig. 10a that EQL method generates large positive residuals at short periods (i.e. $T < 0.3$ s) for motions with PGA >0.4 g, indicating that the EQL method underpredicts ground response at the above ranges. For motions with surface PGA smaller than 0.4 g, the EQL method yields slightly positive residuals (i.e. underpredicion) at short periods. It is worth mentioning that Kaklamanos *et al.* [2013] conducted equivalent linear site response analyses at 100 KiK-net sites, and reported that the EQL method results in an underprediction of ground responses at strains greater than 0.4%, which is very comparable to findings in the current study. Figure 10b shows the computed residuals using the FD-EQL method against recorded data. It can be seen that the FD-EQL method produces negative residuals (approaching −0.3) at short periods, indicating the FD-EQL method yields overpredicted results compared with the observation. It is worth mentioning that above observations are generally consistent with a recent study by Zalachoris and Rathje [2015]. Figure 10c shows the calculated residuals using the M-FD-EQL method with a combination factor of 0.2. The absolute values of residuals are reduced compared with EQL and FD-EQL methods at short periods (i.e. $T < 0.4$ s), indicating that the combination factor of 0.2 provides balanced accuracy benchmarked with downhole array data.

Furthermore, to properly weight earthquake stations with different number of recordings, the concept of mixed-effect regression analysis [e.g. Joyner and Boore, 1993] is used to separate the residuals into intra-site and inter-site components. Given an earthquake event $i$, the residual at site $j$ can be written as the following Eq. (7):

$$r_{ij} = \eta_j + \varepsilon_{ij} \tag{7}$$

where $\eta_j$ denotes the inter-site residual (i.e., average of $r_{ij}$ at site $j$), and $\varepsilon_{ij}$ denotes the intra-site residual. The standard deviation of $\eta_j$ is termed as inter-site standard deviation $\tau$, and the standard deviation of $\varepsilon_{ij}$ is called intra-site standard deviation $\sigma$. The total standard deviation can be computed as $\sigma_{total} = \sqrt{\tau^2 + \sigma^2}$. It is also worth pointing out that the mean of the inter-site residual $\eta_j$ represents the mean bias of the model, since the intra-site residuals $\varepsilon_{ij}$ have a zero mean. To focus on strong motions that may cause damage, this study only quantifies the inter- and intra-site residuals for motions with PGA larger than 0.1 g. Figure 11 summarizes mean of the inter-site residuals, inter-site standard deviation, intra-site standard deviation, and total standard deviation of residuals using three methods. In general, the difference in standard deviation of residuals between different methods is marginal. It can be seen from Fig. 11a that all of the methods overestimate the ground response at periods greater than 0.4 s.

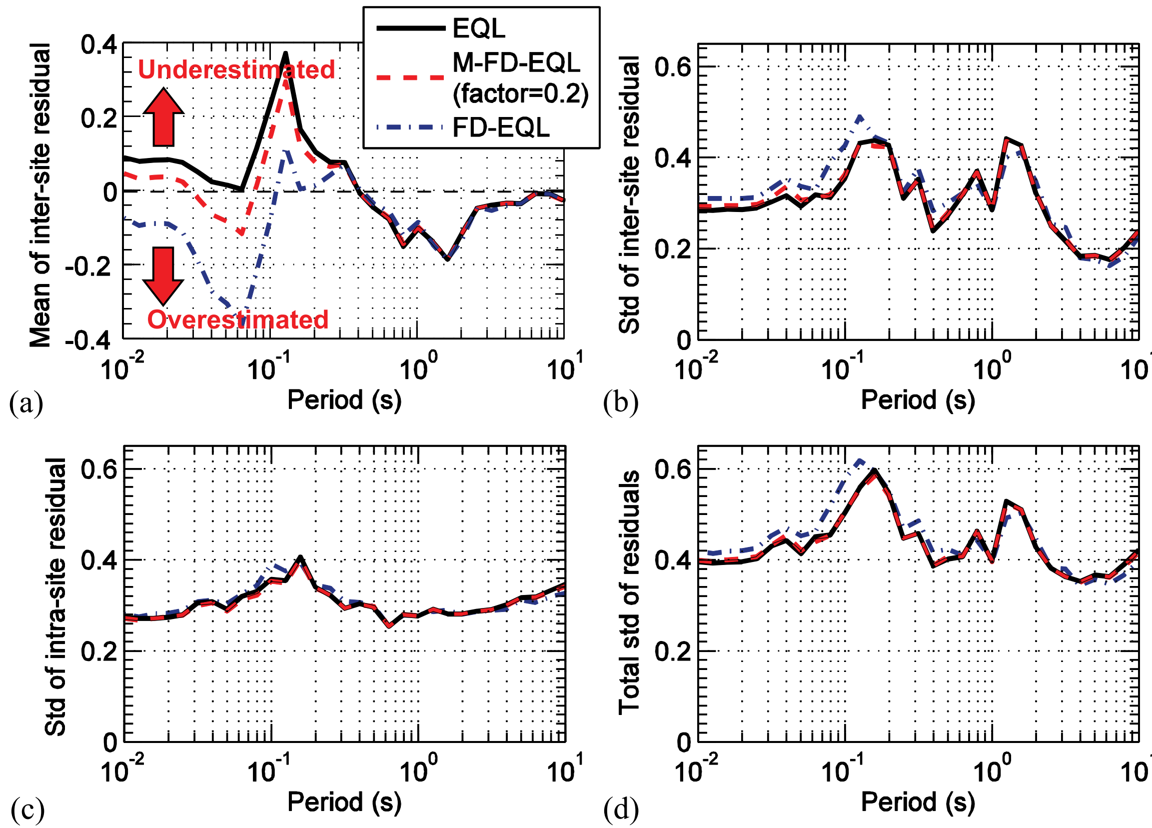



**Figure 11.** (a) Mean of inter-site residual (fixed bias), (b) standard deviation of inter-site residual, (c) standard deviation of intra-site residual, and (d) total standard deviation of residuals (PGA >0.1 g) of LG stations for the EQL method (solid line), the FD-EL method (dotted line), and the M-FD-EQL method (*factor* = 0.2) (dashed line).

The most apparent difference is at the short-period range. It can be observed from Fig. 11a that the EQL method significantly underpredicts the response at spectral periods smaller than 0.2 s, with mean residuals as large as 0.4. Across the same spectral periods, the FD-EQL method overpredicts the amplification with mean residuals approaching −0.4. As discussed earlier, the overprediction at short periods is because the FD-EQL method uses small damping that causes larger response compared with real observation. On the other hand, the proposed M-FD-EQL method (*factor* = 0.2) can further reduce the fixed bias and provide overall better estimates in the short-period range as compared with real observation from downhole arrays. However, inter-/intra-site variabilities would not be significantly affected by using different methods.

Figure 12a summarizes the mean prediction residuals computed using the M-FD-EQL method with a series of combination factors ranging from 0 to 1. To evaluate the effectiveness of combination factors, the following mean squared error (MSE) is defined to quantify the relative difference in fixed bias at multiple periods between estimates and real recordings via Eq. (8):

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}\mu(T_i)^2 \tag{8}$$

where $\mu(T_i)$ denotes the mean of residuals at period $T_i$; $n$ is the number of spectral periods. Figure 12b summarizes MSEs for different combination factors, which is fitted using a polynomial curve. It can be seen that the combination factor of 0.2–0.3 adopted in the M-FD-EQL can be regarded as an optimal value and it yields a MSE close to the minimum. It also worth pointing out that all three methods have similar performance at spectral period larger than 0.3 s, but these responses are slightly overestimated when compared with field observation.

## 4. Conclusions

Although the equivalent linear (EQL) method has been widely used in one-dimensional site response analyses, it somewhat underestimates high-frequency response due to over-damping at these frequency range. On the other hand, the frequency-dependent equivalent linear (FD-EQL) method does enhance high-frequency content of the responses;

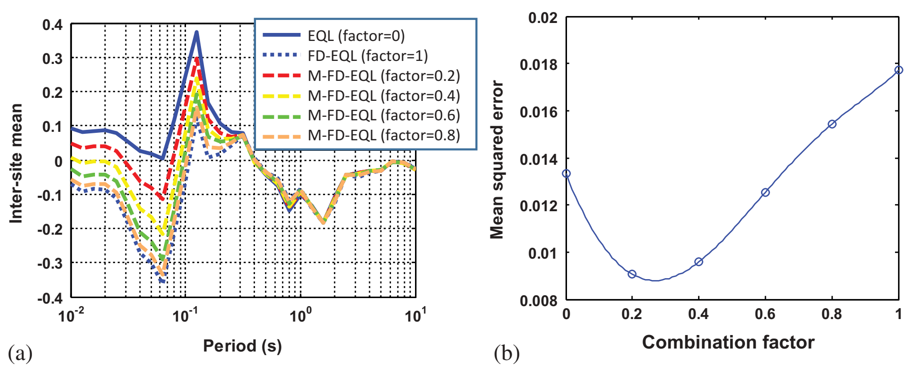



**Figure 12.** (a) Inter-site mean (fixed bias); (b) MSE for different combination factors.

however, they are significantly overestimated based on comparison with NL and real recorded data. The present study proposes a practical scheme (M-FD-EQL) to interpolate the EQL and FD-EQL methods through a combination factor in order to overcome the above limitations. Extensive validation tests have been conducted to benchmark the proposed method with nonlinear analyses of generic NEHRP sites under 996 strong motions. Another validation test was conducted using 383 borehole array data from 16 stations in the KiK-net. The test sites are carefully selected to represent one-dimensional site conditions.

The M-FD-EQL approach provides improvement over the EQL and FD-EQL methods when compared with field measurements from the KiK-net downhole arrays. Based on model validation, an optimal combination factor of 0.2–0.3 in the M-FD-EQL scheme seems to give an overall best result, in particular, for improving the site response spectra at high frequencies (spectral period less than 0.3 s). It is also worth clarifying that the linear combination scheme for the strain spectrum is in a logarithmic scale, so it is not entirely accurate to say the proposed method is a weighted average of the equivalent linear and frequency-dependent equivalent linear methods. The essential scope of the work is for a proper modification of strain spectrum to overcome overamplification of FD-EQL in high frequencies, and the proposed method is just one way to realize such modification.

As a side note, modification made for the M-FD-EQL scheme would not affect the runtime. All these equivalent linear methods (EQL, FD-EQL, M-FD-EQL) only require a few seconds to conduct a site response analysis. The results of this study also indicate that further validation study is needed to address the issues of seismic site response at large strains and at high frequencies.

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