Quantifying epistemic uncertainty and aleatory variability of Newmark displacements under scenario earthquakes

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ABSTRACT: Earthquake-induced slope displacement is an important parameter for safety evaluation and design of slope systems. Many Newmark displacement models have been recently developed using various ground motion databases and different intensity measures as predictors. In this study, the epistemic uncertainties among different models and aleatory variability of the predicted Newmark displacements are quantified. The epistemic uncertainty is represented by the variation between different model predictions. The standard deviation of the uncertainty is approximately 0.5-1 in natural log scale for scenario earthquakes of Mw 5.5-7.5, which is much larger than that of the ground motion prediction equations. This indicates further development of the Newmark displacement models is much needed. The total aleatory variability considering both GMPEs and displacement models is roughly 1.5-2.5 in natural log scale. The large epistemic uncertainty and aleatory variability imply that it is extremely important to account for both in seismic slope analysis.

1 INTRODUCTION

Newmark displacement model is commonly used to estimate the seismic performance of slopes during earthquake (Newmark, 1965). The Newmark displacement model assumes the sliding mass is rigid, and sliding occurs on a predefined interface. The critical acceleration (a_c) represents the resistance of the slope against sliding. It can be determined by the strength of material and the slope angle etc. Sliding is initialized when the shaking acceleration exceeds the critical acceleration, and the block displaces plastically along the interface. The permanent displacement D is calculated by double integrating the exceeded accelerations with respect to time (Figure 1). Although the simple rigid-plastic model does not consider the deformation of the block itself during shaking, this method has been widely used to evaluate earthquake-induced displacement for natural slopes (Jibson 2007).

Empirical equation to estimate the Newmark displacement was first proposed by Ambraseys & Menu (1988) as a function of the critical acceleration ratio (critical acceleration a_c over the peak ground acceleration PGA). Throughout years, many researchers (e.g., Jibson, 2007; Saygili & Rathje, 2008; Hsieh & Lee, 2011) have proposed various empirical equations using various ground motion intensity measures (IMs) as predictors, including peak ground acceleration (PGA), moment magnitude of the earthquake (M), peak ground velocity (PGV) *etc.* Arias intensity (I_a) is also an important predictors, as it incorporates cumulative effect of an acceleration time history and represent a measure of earthquake energy. Arias Intensity is calculated by the following equation (Travasarou & Bray, 2003):

$$Ia = \frac{\pi}{2g}\int_0^{t_{tot}} a(t)^2\, dt \qquad (1)$$

where g is the acceleration of gravity, $a(t)$ is the acceleration time history, and t_{tot} is the total duration of the time history.

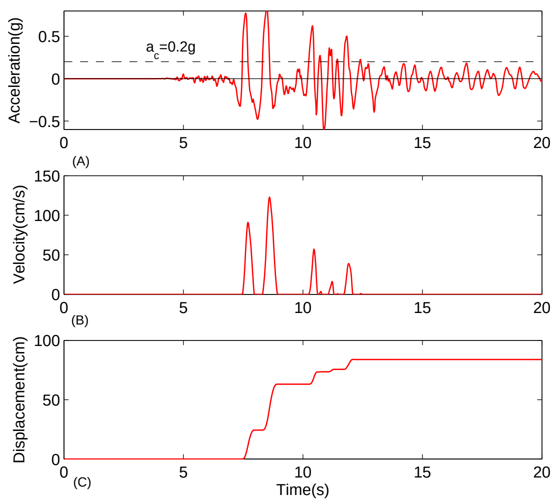


Figure 1. Illustration of Newmark displacement with critical acceleration a_c=0.2g. (A) Earthquake acceleration-time history. (B) Velocity of sliding block versus time. (C) Displacement of sliding block versus time.

Because different database and functional forms are used in developing these empirical prediction models, it is not surprising that these models would yield different predicted results. Uncertainty associated with the Newmark displacement models can be classified as two categories: epistemic uncertainty and aleatory uncertainty. Epistemic uncertainty is due to lack of knowledge. In principal, it can be reduced by using sufficient data or improved regression techniques. The epistemic uncertainty can be approximately evaluated by the variation of different model predictions. Aleatory variability, on the other hand, represents inherent randomness that can not be reduced. Aleatory variability is usually quantified by variation of the observed data against the model prediction. Recently, Douglas (2010, 2012) studied the consistency of ground motion prediction equations (GMPEs) developed in the past four decades for IMs such as PGA, PGV, Ia, etc. However, the epistemic and aleatory uncertainties of the Newmark displacement models have not yet been thoroughly investigated.

This study aims at quantifying the epistemic uncertainty and aleatory variability of Newmark displacement models. Ten recently developed Newmark displacement models are used in this study and listed as follows:

(1) [PGA] RS08 model (Saygili & Rathje, 2008):

$$\ln D = 5.52 - 4.43\left(\frac{a_c}{PGA}\right) - 20.39\left(\frac{a_c}{PGA}\right)^2 + 42.61\left(\frac{a_c}{PGA}\right)^3 - 28.74\left(\frac{a_c}{PGA}\right)^4 + 0.72\ln(PGA)$$

$$\sigma_{\ln D} = 1.13 \tag{2}$$

(2) [PGA, Ia] RS08 model (Saygili & Rathje, 2008):

$$\ln(D) = 2.39 - 5.24\left(\frac{a_c}{PGA}\right) - 18.78\left(\frac{a_c}{PGA}\right)^2 + 42.01\left(\frac{a_c}{PGA}\right)^3 - 29.15\left(\frac{a_c}{PGA}\right)^4 - 1.56\ln(PGA) + 1.38\ln(Ia)$$

$$\sigma_{\ln D} = 0.46 + 0.56\left(a_c / PGA\right) \tag{3}$$

(3) [PGA, PGV] RS08 model (Saygili & Rathje 2008):

$$\ln D = -1.56 - 4.58\left(\frac{a_c}{PGA}\right) - 20.84\left(\frac{a_c}{PGA}\right)^2 + 44.75\left(\frac{a_c}{PGA}\right)^3 - 30.5\left(\frac{a_c}{PGA}\right)^4 - 0.64\ln(PGA) + 1.55\ln(PGV)$$

$$\sigma_{\ln D} = 0.41 + 0.52\left(a_c / PGA\right) \tag{4}$$

(4) [PGA, Ia, PGV] RS08 model (Saygili & Rathje 2008):

$$\ln(D) = -0.74 - 4.93\left(\frac{a_c}{PGA}\right) - 19.91\left(\frac{a_c}{PGA}\right)^2 + 43.75\left(\frac{a_c}{PGA}\right)^3 - 30.12\left(\frac{a_c}{PGA}\right)^4 - 1.3\ln(PGA) + 1.04\ln(PGV) + 0.67\ln(Ia)$$

$$\sigma_{\ln D} = 0.2 + 0.79\left(a_c / PGA\right) \tag{5}$$

(5) [PGA, M] RS09 model (Rathje & Saygili, 2009):

$$\ln(D) = 4.89 - 4.85\left(\frac{a_c}{PGA}\right) - 19.64\left(\frac{a_c}{PGA}\right)^2 + 42.49\left(\frac{a_c}{PGA}\right)^3 - 29.06\left(\frac{a_c}{PGA}\right)^4 + 0.72\ln(PGA) + 0.89(M_w - 7)$$

$$\sigma_{\ln D} = 0.73 + 0.79\left(a_c / PGA\right) - 0.54\left(a_c / PGA\right)^2 \tag{6}$$

(6) [PGA] J07 model (Jibson, 2007):

$$\log_{10}(D) = 0.215 + 2.341\log_{10}\left(1 - \frac{a_c}{PGA}\right) - 1.438\log_{10}\left(\frac{a_c}{PGA}\right)$$

$$\sigma_{\log_{10} D} = 0.51 \tag{7}$$

(7) [PGA, M] J07 model (Jibson, 2007):

$$\log_{10}(D) = -2.71 + 2.335\log_{10}\left(1 - \frac{a_c}{PGA}\right) - 1.478\log_{10}\left(\frac{a_c}{PGA}\right) + 0.424\,\mathrm{M}_w$$

$$\sigma_{\log_{10} D} = 0.454 \tag{8}$$

(8) [Ia] J07 model (Jibson, 2007):

$$\log_{10}(D) = 2.401\log_{10}(Ia) - 3.481\log_{10}(a_c) - 3.23$$

$$\sigma_{\log_{10} D} = 0.656 \tag{9}$$

(9) [PGA, Ia] J07 model (Jibson, 2007):

$$\log_{10}(D) = 0.561\log_{10}(Ia) - 3.833\log_{10}\left(\frac{a_c}{PGA}\right) - 1.474$$

$$\sigma_{\log_{10} D} = 0.616 \tag{10}$$

(10) [Ia] HL11 model (Hsieh and Lee, 2011):

$$\log_{10}(D) = 0.847\log_{10}(Ia) - 10.62a_c + 6.587a_c\log_{10}(Ia) + 1.84$$

$$\sigma_{\log_{10} D} = 0.295 \tag{11}$$

Among these equations, the unit of the Newmark displacement *D* is cm; *PGA* and a_c are in the unit of g; *PGV* is in the unit of cm/s; and I_a is in the unit of m/s. [PGA] RS08, [PGA] J07, [Ia] J07, [Ia] HL11 models employs only a single IM as the predictor. Therefore, they are called scalar models. The other models employ a combination of more than one IMs (PGA, Ia or PGV), and they are called vector models.

2 EPISTEMIC UNCERTAINTY OF NEWMARK DISPLACEMENT MODELS

In this study, epistemic uncertainty of the above Newmark displacement models will be studied. Earthquake scenarios considered in the analysis are moment magnitude (M_w) 5.5, 6.5 and 7.5 events on a strike-slip fault. The site condition is assumed to be a stiff soil site with V_{s30}=400 m/s. Four Next Generation Attenuation (NGA) GMPEs are used to predict PGA and PGV (Abrahamson & Silva, 2008; Boore & Atkinson, 2008; Campbell & Bozorgnia, 2008; Chiou & Youngs, 2008). The GMPEs proposed by Travasarou & Bray (2003), Foulser-Piggott & Stafford (2012) and Campbell & Bozorgnia (2012) are chosen for predicting Ia. In order to minimize the influence of epistemic uncertainty existed in the GMPEs, the predicted median IMs are averaged as the input to predict the median Newmark displacements. By this way, the epistemic uncertainty and aleatory variability in the GMPEs are neglected.

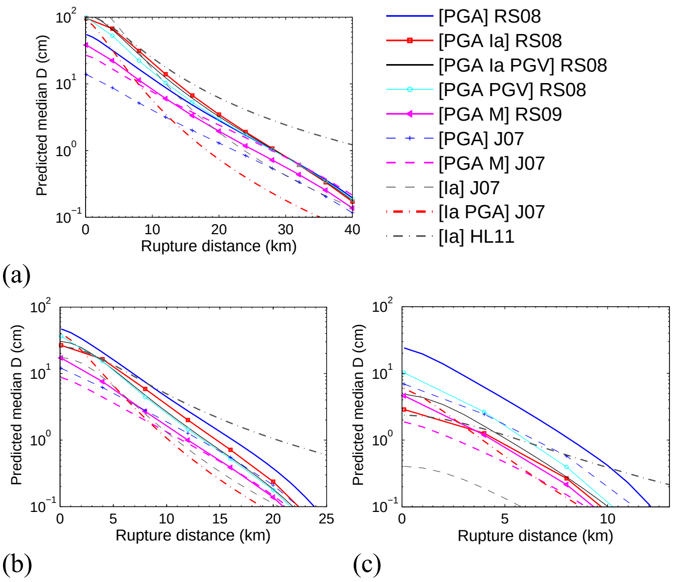


Figure 2. Median predicted displacements by various Newmark displacement models for (a) M_w 7.5 (b) M_w 6.5 (c) M_w 5.5 earthquake on a strike-slip fault: Note: for each scenario, the average of the predicted median IMs are used as input parameters.

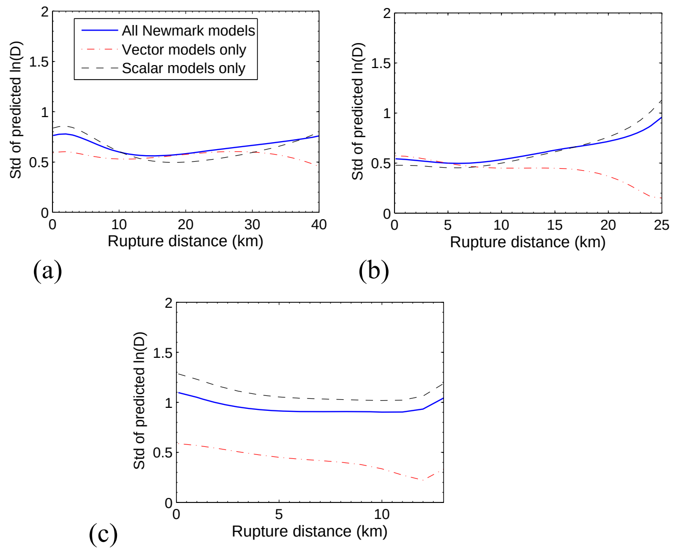


Figure 3. Standard deviations of the median Newmark displacements (in natural log scale) predicted by the ten Newmark displacement models: (a) For a M_w 7.5 strike-slip earthquake; (b) For a M_w 6.5 strike-slip earthquake; (a) For a M_w 5.5 strike-slip earthquake. Same as Figure 2, the average of the median predicted IMs by various GMPEs are used as input IMs.

Figure 2 shows the predicted median sliding displacements versus rupture distances assuming a_c=0.1g and M_w equals 7.5, 6.5 and 5.5, respectively. It can be observed that the predicted displacements by different Newmark displacement models vary significantly for a given scenario. For example, the estimated median displacement ranges from 4 cm to 24.6 cm for M_w 7.5 at the rupture distance of 10 km. As rupture distance increases, all the predicted sliding displacement decreases as expected. Epistemic uncertainty is expected to be higher if the predicted values among these models are more scattered. From the figures, the predicted displacements (in log scale) are more scattered as rupture distance increases, especially for smaller magnitude events. This is mainly due to specific functional forms chosen by different models. Some models decays much faster as rupture distance increases. Since very small displacement values are not of engineering significance, it is more rational to focus only on large displacement amplitudes. Figure 3 shows the standard deviations of the predicted median displacements by these models for three scenarios. The standard deviations among predicted values are in the range of 0.5-1 (in natual log scale) for all three scenarios. The M_w 6.5 scenario shows the smallest epistemic uncertainty and the M_w 5.5 event has the largest uncertainty, which is constantly greater than 1 if all Newmark displacement models are considered. This is not unexpected since that the number of larger magnitude events (M_w>6) is usually dominant in the ground motion databases that are used to develop the Newmark displacement models. In addition, the epistemic uncertainty among six vector models are much smaller than that of four scalar models for all cases considered, as shown in Figure 3. In general, the standard deviation of vector models is in the range of 0.4-0.6 (in natural log scale) for various scenarios. This reveals that the vector models can provide more consistent results by using multiple IMs. The vector IMs represent different aspects of ground motion properties and provide more information about the ground motion characteristics.

Furthermore, epistemic uncertainties of both GMPEs and Newmark displacement models are considered in the following analysis. Instead of applying the averaged median IMs to compute sliding displacements as before, the predicted median IMs are chosen from different GMPEs (one from four NGA models for PGA, PGV, and one from three GMPEs for Ia) to compute the median sliding displacement using 10 Newmark displacement model, as shown in Figure 4. All together, there are 120 (4×3×10) combinations of displacements in each subplot. The displacements are hugely scattered, indicating that selection of different GMPEs and Newmark displacement models would significantly influence the final results. For example, the estimated median displacement ranges from 2 cm to 60 cm for M_w 7.5 at the rupture distance of 10km. Figure 5 shows the standard deviations of displacements (in natural log scale) shown in Figure 4. The standard deviations by considering both epistemic uncertainties existed in GMPEs and Newmark displacement models is generally 20% larger then the previous analysis by only considering the epistemic uncertainty of the Newmark displacement models.

The standard deviation obtained from the latter is generally 20% larger than the former case. The results are consistent with recent studies on the epistemic uncertainty of IMs. The reported standard deviations of the epistemic uncertainty for PGA, PGV and Ia are approximately 0.2-0.4 in the natural log scale (Douglas, 2010; 2012). The epistemic uncertainty of the Newmark displacement models appears to be much larger than that of the GMPEs, probably due to inherent difficulty to correlate the sliding displacements with IMs using simple function forms. It calls for more research efforts to be placed to develop more advanced Newmark displacement models in the future

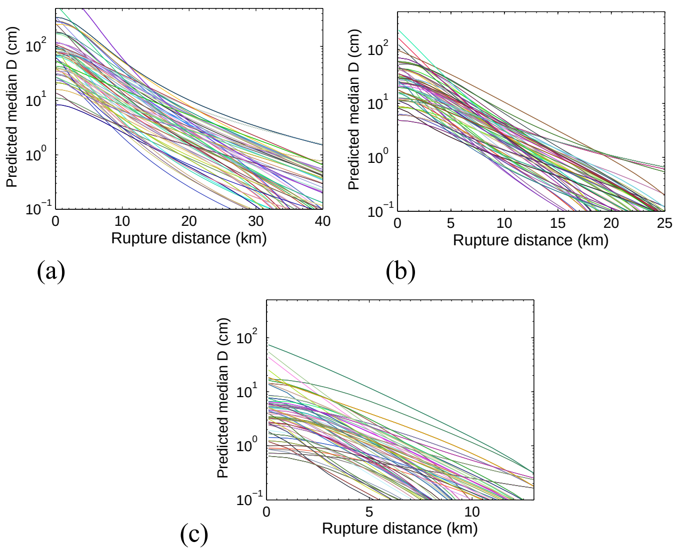


Figure 4. Variation of the displacements predicted by various GMPEs and Newmark displacement models, for (a) M_w 7.5 (b) M_w 6.5 (c) M_w 5.5 strike-slip earthquakes.

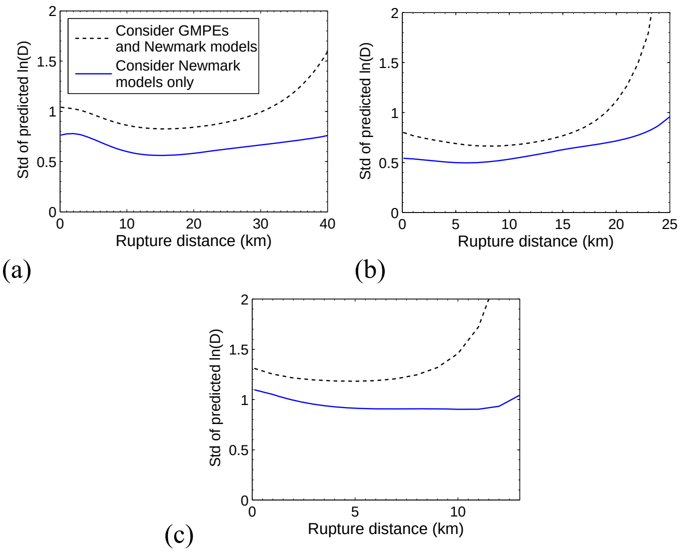


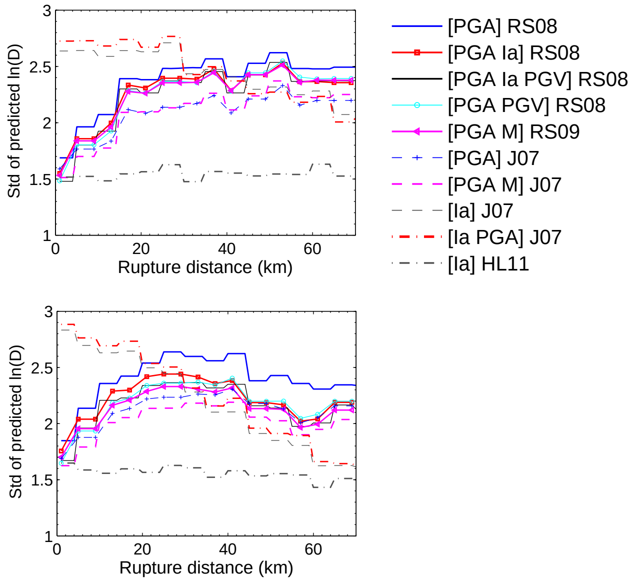
Figure 5. Standard deviations of predicted displacements in natural log scale, considering epistemic uncertainty of both GPMEs and Nemwark displacement models, or considering only epistemic uncertainty of Newmark displacement models, for (a) M_w 7.5 (b) M_w 6.5 (c) M_w 5.5 strike-slip earthquakes.

3 ALEATORY VARIABILITY OF NEWMARK DISPLACEMENT MODELS

The aleatory variability of Newmark displacement models is also compared for given earthquake scenarios. Although vector models usually reported a smaller standard deviation compared with the scalar models by incorporating more predictors, inclusion of additional IMs may also induce extra variability in the IMs themselves. Therefore, the total aleatory variability of the Newmark displacement for a scenario earthquake should count for contributions from aleatory variability in GMPEs and that in Newmark ground motion models.

Monte-Carlo simulation is used to evaluate the total aleatory variability of the Newmark displacement for a given scenario earthquake. First, 100 sets of correlated vector IMs are generated for a specific scenario. The vector IMs are assumed to follow multivariate lognormal distribution with mean and standard deviation specified by GMPEs. For vector models, the joint occurrence of multiple IMs is specified by the empirical correlations between them. The correlation coefficients among PGA, PGV and Ia are specified as ρ(PGA, Ia)=0.88, ρ(PGA, PGV)=0.69 and ρ(Ia, PGV)=0.74 (Campbell and Bozorgnia, 2012), respectively. Secondly, for each set of vector IMs, 100 Newmark displacement values are simulated by assuming the displacements follow a lognormal distribution. The standard deviation of the resulted 10000 displacement values is then calculated to estimate the total aleatory variability for each Newmark displacement model. It is noted that very small displacement values have to be excluded, since they are of little engineering importance but appear to be highly scattered in log scale. Figure 6 shows the obtained total standard deviations versus rupture distances for different Newmark displacement models by only considering displacement values greater than 0.001 cm. Quite similar trend can be observed for both magnitudes considered. Generally speaking, the total standard deviations (in natural log scale) fall in the range of 1.5-2.5 for different models, which implies that the displacement distribution is significantly scattered for each scenario. For example, although the reported standard deviation is 0.295 in $\log_{10}$ scale (0.68 in natural log scale) for the [Ia] HL11 model, the total standard deviation of displacements is as high as 1.5 in natural log scale, due to large standard deviation of Ia ($\sigma_{\ln Ia}\approx 1$).

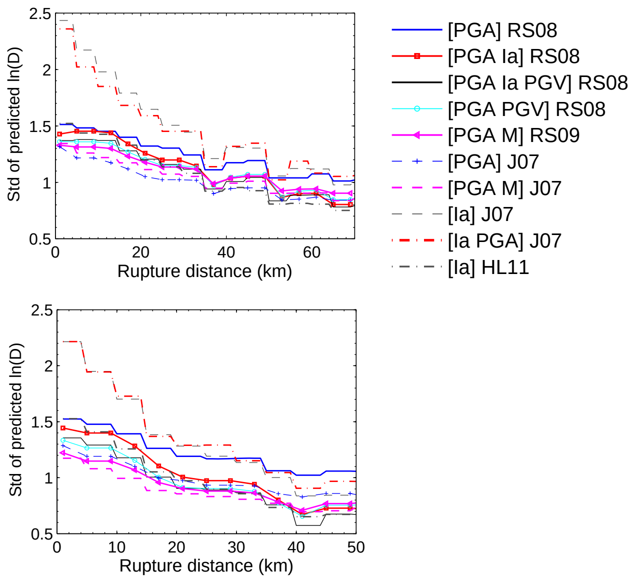
If the cutoff displacement value is set as 1 cm, which is the recommended upper bound for negligible displacement by Bray & Travasarou (2007), the total standard deviations versus rupture distances are shown as Figure 7. In this case, the standard deviations are generally smaller compared with the values shown in Figure 6. This is expected since the scatter of larger displacement values would be inevitably smaller in the log scale. The standard deviations decreases as separation distance increases, which are approximately within the range of 1-1.5 for most models. The total aleatory variability appears to be rather consistent for all vector models and scalar models. This is a favorable result since the aleatory variability is the inherent randomness. In principle, it should not change significantly from model to model.



(a)

(b)

Figure 6. Standard deviations considering aleatory variability of GMPEs and Newmark displacement models, for (a) M_w 7.5 (b) M_w 6.5 strike-slip earthquakes. Cutoff displacement value is 0.001 cm.



(a)

(b)

Figure 7. Standard deviations considering aleatory variability of GMPEs and Newmark displacement models, for (a) M_w 7.5 (b) M_w 6.5 strike-slip earthquakes. Cutoff displacement value is 1 cm.

4 CONCLUSIONS

In this paper, epistemic uncertainty and aleatory variability of Newmark displacement models are quantified for given scenario earthquakes. Since the Newmark displacement models are based on IMs (e.g., PGA, Ia) as predictors, the uncertainties to predict these IMs have to be considered in the analysis. The standard deviation of the epistemic uncertainty of the Newmark displacement models is within the range of 0.5-1 using the predicted median IMs as input parameters. In general, the standard deviation of epistemic uncertainty becomes 20% larger if both of the epistemic uncertainties of GMPEs and Newmark displacement models are considered. The epistemic uncertainty in the Newmark displacement is significant larger compared with that of GMPEs. It implies that it is more efforts should be placed to develop advanced Newmark displacement models to reduce the epistemic uncertainty in the future.

The total aleatory variability of the Newmark displacement model is also studied for given scenario earthquakes. Choosing different cutoff displacement values would result in different total aleatory varialibty. Considering both aleatory variailibilities of GMPEs and the Newmark displacement models, the total standard deviation of predicted displacement are 1.5-2.5 in natural log scale if the cutoff value is chosen as 0.001cm. The total aleatory variability will be reduced if the cutoff value is chosen as 1 cm. The total aleatory variabilities are rather consistent for scalar models and vector models considered.

It is important to emphasize that vector models do not significantly reduce the total aleatory variability of the predicted displacements, due to additional sources of variability introduced by incorporating additional IMs. However, the epistemic uncertainty for vector models is generally smaller than that of the scalar models. The epistemic uncertainty of vector models tends to be consistent for all scenarios considered, since incorporating multiple IMs can better satisfy the sufficiency criterion (Saygili & Rathje, 2008), such that the model is not biased for different magnitude and distances.

In current practice, seismic slope analysis is a two-step process. First, the ground motion IMs (e.g. PGA, PGV or Ia) are estimated using GMPEs; Secondly, the estimated IMs are used as input parameters to the Newmark displacement models. Both of the epistemic uncertainties and aleatory variabilities in GEMPs and Newmark displacement models should be well considered in seismic hazard analysis of slopes. A reasonable logic tree analysis should be employed to account for epistemic uncertainties in GMPEs and Newmark displacement models. It is also envisioned that developing Newmark displacement models directly based on earthquake scenarios (magnitude and distance etc.) would reduce the inherent complexity and uncertainties in current two-step approach.

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